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A MANUAL

OF

PHYSICAL MEASUREMENTS

BY

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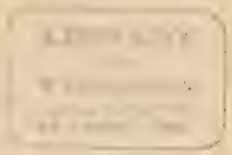
FOURTH EDITION. REVISED AND ENLARGED



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PREFACE.

This manual has been prepared to meet the needs of students beginning work in the Physical Laboratory of the University of Michigan. Such a book must inevitably possess a certain local coloring peculiar to the conditions it has been designed to meet. A manual equally suited to all laboratories, has not been and probably will not be written. Each laboratory reflects in greater or less degree the individual trend of the man who stands at its head, and its exercises and methods are the result of an extended process of adaptation and assimilation. Hence it happens that one laboratory is largely devoted to the study of the phenomena of light, another to those of electricity, and a third to those of elasticity, heat, or electrochemistry, as the case may be. The moral of all this is, that the practice and traditions of each laboratory are best conserved by a text representative of its own methods, and if no better reason should be found, perhaps this may serve to explain the appearance of this, another laboratory manual.

The exercises herein described embody the work required of students in Physics and in Engineering in their first course in Physical Laboratory Practice. Such a course is expected to occupy three laboratory periods of two hours each for one semester, and embraces some thirty-six to forty of the exercises in this manual. Owing to the diversity of the work prescribed in the various courses in Engineering, no one student is expected to complete all the exercises in this book in a single semester.

In accordance with the practice in the University of Michigan, it is expected that the laboratory work shall be supplemented by lectures upon the theory of the exercises, and recitations upon the work actually done and the results obtained. In this way it is believed that the student is brought to a clearer understanding of the significance of the exercise and of the accuracy attainable

under given conditions. To this end the exercises are numbered consecutively throughout the text, and those under any specific subject are preceded by sufficient theory to render the formulæ and methods clear to persons familiar with the fundamental principles of Physics as set forth in any standard textbook.

Being designed for beginners in the Physical Laboratory, this manual makes no claim to completeness, either in subject matter or in exposition. The aim has been to furnish a coherent and logical series of graded exercises in Physical Measurement, such as will best furnish an introduction to Practical Physics, and at the same time afford opportunity for developing ability in recording and interpreting observations, and skill in the manipulation of delicate and sensitive apparatus.

For convenience of reference a series of tables of the more important physical constants, of squares, cubes, square roots and multiples of π, of the logarithms of numbers, and the trigonometric functions have been added. A thorough drill in the use of logarithmic tables in the computation of results, should form a feature of any successful course in Laboratory Practice. To this end an orderly method of procedure in such computation has at all times been insisted upon.

The authors have drawn freely from many standard works on Practical Physics, notably from those of Kohlrausch, and Stewart and Gee in General Physics, and from Carhart and Patterson's Electrical Measurements.

In conclusion we wish to thank our colleagues, Professors Carhart and Patterson, for helpful suggestions and criticisms during the preparation of the work.

University of Michigan, March, 1902.

FROM THE PREFACE TO THE SECOND EDITION.

The necessity for a second edition of this book has presented an opportunity for a careful revision of the text, both in the elimination of errors and in the addition of certain features which experience has shown to be desirable and necessary to make the manual truly representative of modern laboratory practice. In making these additions the needs of the average student have been kept constantly in mind both as regards his previous preparation and the requirements laid upon him by his subsequent University work.

In the first part several articles are devoted to the measurement of angles, and a chapter has been added upon Surface Tension and Viscosity. In order to meet more fully the demands made upon students of Mechanical and Electrical Engineering the chapters upon Heat and Electricity have been practically rewritten. Several of the articles in these chapters contain new and important matter, notable among which are the discussion of the ballistic d'Arsonval galvanometer, and the exercises involving the use of the potentiometer and of the thermoelement.

While the additions have in general been such as to render the work more advanced in character, with the possible exception of some exercises in the measurement of angles, still it is hoped that the book will not be found less useful for elementary work than before. The forms for recording results and the outlines for computation have abundantly justified the wisdom of their insertion in the immense saving of time and energy to the busy instructor. While it has been urged by some that students readily and intuitively devise explicit, symmetrical and logical arrangements for their data and computations, such students have as yet entirely escaped our observation.

September, 1906.

PREFACE TO THE THIRD EDITION.

The third edition of this manual has been prepared mainly because it was felt that the arrangement of the subject matter should correspond more closely to that found in the authors' COLLEGE PHYSICS, recently published by the Macmillan Company.

The book has also been thoroughly revised, the treatment been changed in a number of places and a few exercises, notably some elementary exercises in electricity, been added.

The authors are greatly indebted to their colleagues, Professor H. M. Randall and Mr. W. W. Sleator, for assistance in reading the proof sheets of the present edition.

JOHN O. REED.
KARL E. GUTHE.

August, 1912.

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PHYSICAL MEASUREMENTS

INTRODUCTION

1. **Benefits of Laboratory Work.** The benefits to be derived by the student from work in the physical laboratory are two-fold. In the first place he is to become acquainted with delicate instruments, to be trained to make systematic, accurate and independent observations, and to compute from the data so obtained, the values of many of the more important physical constants. Secondly, he is to make a searching review of the fundamental principles of the science, and to be brought to a more lively realization of the meaning and importance of formulae and laws deduced in the text-books upon general physics. To this end it is urgently advised that the student familiarize himself with all the details of the theory of the experiment, and **be able to sketch from memory** the apparatus to be employed before beginning any experiment. An attempt to follow directions but dimly understood, and to manipulate apparatus whose construction and purpose are alike unknown, can only result in loss of time, and laboratory work under such circumstances is practically worthless as a means of discipline.

2. **Instruments.** The instruments used in the physical laboratory are usually of delicate construction; many of them are costly and liable to injury from rough or careless usage. It is of the highest importance that all apparatus should be handled with care, and returned to its proper place after use. If any piece is found to be out of adjustment or in need of repair, report the fact

before beginning work. If any screws or other parts of an instrument do not move readily, do not apply force but report the matter to the instructor in charge. The ability to use delicate apparatus without injuring or destroying it is an important part of a liberal education.

3. **Record of Observations.** All observations, data and necessary formulae, such as the time and place of the exercise, the specific instruments used, the objects measured, etc., are to be recorded in a note book provided for that purpose. Such a book should have **fixed leaves,** and be made of paper suitable for writing with ink. The record is to be made **at the time of the experiment.** It must contain the detailed information necessary, must be clearly written and arranged in a neat, methodical manner, so that one familiar with the experiment may readily comprehend what has been done. It should be sufficiently specific to be intelligible after the circumstances of the experiment are entirely forgotten.

A suitable form of record has been appended to each exercise in the manual, and the student will do well to follow these, at least until he is able to arrange the material for himself. Tabulation of results in columns adds much to the neatness of a note book and materially assists in the detection of errors, either of record or of observation. It is suggested that the note books be casually inspected at each exercise if possible, by the instructor, in his rounds in the laboratory.

The computation of results should generally be done at home, and a record of the complete experiment be handed to the instructor for approval. In this report the date, number, name and object of the exercise should precede a short discussion, derivation of formulae, the record of observations and the computed results. If these reports be written on loose sheets to be returned to the student after approval by the instructor, these sheets must be kept, ready for inspection, in a suitable binder. A separate page should always be begun for each new experiment.

4. **Graphical Methods.** It is frequently of interest to verify a law, stating the relation that is known to exist between two

quantities, or to detect and determine such a relation where it is not known. In all such cases the results obtained by observation are most clearly presented to the eye when plotted as a curve. In the application of the method it is customary to plot values of the independent variable as abscissae and those of the dependent variable as ordinates. In all cases where the phenomenon under investigation is continuous, a smooth curve sketched through the points obtained, may be assumed to represent the facts better than any individual observation. The graphical method has the additional advantage that an accidental error is at once made evident by the fact that **the point so obtained departs markedly from the curve.**

The curve most readily plotted and tested is **the straight line,** and it is advisable so to transform the assumed relation as to render the plotting of a straight line practicable.

For example, suppose it were desired to investigate the relation between the time of vibration of a pendulum and its length. If we assume that this relation may be expressed by an algebraic function of the form

$$T = a\, l^m \qquad (1)$$

we may determine the constants a and m from a series of observations. Passing to logarithms we have

$$\log T = m \log l + \log a \qquad (2)$$

This is clearly of the form

$$y = m\, x + c \qquad (3)$$

and is therefore the equation of a straight line. If now we plot values of $\log l$ as abscissae and the corresponding values of $\log T$ as ordinates, we may decide at once whether such a relation as we

have assumed exists, and we may obtain values of a and m directly from the curve.

It is not necessary that the same numerical value should be assigned to a scale division on the horizontal and vertical axes. In general it is best to make the value of a scale division correspond, as nearly as possible, to the least quantity which we can measure. If this be impossible, such values should be chosen for a scale division on each axis as will cause the curve most nearly to fill the page with the observations to be plotted. It is only in case we wish to determine from the curve a quantity which is represented by the tangent of an angle, *i. e.,* which represents the ratio between the coördinates, that it is necessary to assign to them their proper relative values.

A table of the data from which the curve has been plotted **should in all cases accompany the curve** and **the values assigned to a scale division** on the horizontal and vertical axes **must be clearly stated upon these axes.** In practice it is well to prick with a needle the exact position of each point on the curve and then draw around each point a small circle in colored ink. All curves should be plotted upon special cross-section paper, drawn in ink, and the points clearly marked as indicated above. In case special accuracy is desired it is well to use paper printed from engraved plates. The curves are to be inserted in the note book **after the record of the experiment.** This is most readily done by cutting away about two-thirds of a sheet lengthwise, and pasting the stub to the back of the cross-section paper.

The following data may be used as exercises in the plotting of curves. In case any set of data does **not** represent **continuous phenomena** how should the curve be drawn?

I. POPULATION OF THE UNITED STATES.

Year	Population	Year	Population
1790	3 929 214	1860	31 443 321
1800	5 308 483	1870	38 558 371
1810	7 239 881	1880	50 155 783
1820	9 633 822	1890	62 622 250
1830	12 866 020	1900	76 303 387
1840	17 069 453	1910	93 402 151
1850	23 191 876		

II. ATTENDANCE AT THE UNIVERSITY OF MICHIGAN.

Collegiate Year	No. of Students	Collegiate Year	No. of Students
1881-82	1534	1897-98	3223
1882-83	1440	1898-99	3192
1883-84	1377	1899-1900	3441
1884-85	1295	1900-1901	3712
1885-86	1401	1901-1902	3709
1886-87	1572	1902-1903	3792
1887-88	1667	1903-1904	3957
1888-89	1882	1904-1905	4136
1889-90	2153	1905-1906	4571
1890-91	2420	1906-1907	4746
1891-92	2692	1907-1908	5010
1892-93	2778	1908-1909	5223
1893-94	2659	1909-1910	5383
1894-95	2864	1910-1911	5381
1895-96	3014	1911-1912	5582
1896-97	2975		

III. HYSTERESIS CURVE FOR SWEDISH IRON.

The values are given for half a cycle only. To obtain the complete curve, reverse the signs of both columns.

Intensity of Field H	Magnetic Induction B
0	— 8300
1.5	— 5816
2.7	0
3.7	+ 3041
5.0	+ 5284
6.6	+ 7037
8.8	+ 8658
11.3	+ 9923
16.0	+ 11389
25.2	+ 12898
36.2	+ 13808
45.9	+ 14430
62.4	+ 15074
45.0	+ 14652
21.1	+ 13653
6.6	+ 11766
4.6	+ 11122
0	+ 8300

5. **Errors of Observation.** In any series of measurements of the same physical quantity, we find that the results differ slightly from one another, owing to imperfections of the instrument or errors in making the readings. These errors are not to be con-

founded with mistakes in calculation or errors in recording observations; such errors must of course be rejected. First to be considered are the accidental errors of observation which, if the observations have been made without any bias, or preconceived idea as to what the value **ought to be,** are likely to be positive as often as negative and may, in the long run, be considered as having little influence upon the mean result. Obviously the influence of such errors is diminished by making the number of observations as large as possible and taking the arithmetical mean. The latter may be regarded as an observation made with a more precise instrument. The average of n observations has a measure of precision $\sqrt{n}$ times as large as that of a single observation. *i. e.,* if we take the mean of sixteen observations, then the probable error of this mean is only one-fourth of that of a single observation.

It is not, however, at all times convenient nor necessary to multiply the number of observations. If a set of readings differ but little among themselves, it is clear that little will be gained by increasing the number of such observations. On the other hand if the individual readings differ markedly among themselves, a much larger number must be taken if the average reading is to be considered as trustworthy. The conditions of the experiment and the degree of accuracy desired, must in all cases be the determining factors.

6. **Probable Error.** What is more to the purpose, is to give a clear idea of the degree of precision attained in a given measurement. This is done by computing what is known as the **probable error** of the mean result. The computation is made by writing the readings in a vertical column, and placing opposite each reading the difference between it and the mean, making it plus or minus, according as the reading is greater or less than the mean. This difference is termed the "residual" for the observation in question. It is shown in the theory of probabilities that the probable error of the mean of n observations, is

$$E = \pm 0.6745 \sqrt{\frac{s}{n\,(n-1)}} \qquad (4)$$

where s is the sum of the squares of the residuals of the n observations. The probable error is not the *most* probable error, nor is it a limiting value. It is that error which is just as likely to be exceeded as not, or it is an **even chance** that the **error is smaller,** as that it is **larger** than the probable error.

In addition to accidental errors of observation, there are to be considered the errors arising from faults in the instrument or in the method of observation. These are classed as **systematic errors** and are neither to be eliminated nor estimated by computation. They cannot be removed entirely, but may be minimized by repeated measurements with different instruments in the hands of different observers. Much thought should therefore be expended upon devising correct methods of observation, and means for avoiding systematic errors, since upon these the accuracy of the result must finally depend. Students are to be urged to use judgment in measurements and warned against excessive care in guarding against minute mistakes, while errors of the grossest kind are liable to be made in the process. It frequently occurs that a student in measuring the length of a wire will expend much time in determining the length to hundredths of a millimeter, and yet make an error of a centimeter in the result.

7. **Influence of Errors upon the Result.** It is frequently necessary to compute a result from one or more quantities obtained by observation. In such cases it is of interest to determine the influence of errors in the observed quantities upon the computed result. If X be the value sought, and x the value of the quantity observed, then X is some function of x. If e be the error in x due to all causes, and E the error in X consequent upon e, then

$$X + E = f(x + e). \tag{5}$$

It may be shown that the total error of the final result is approximately

$$E = e \frac{dX}{dx} \tag{6}$$

If E_1 be the error in the derived result arising from an error e_1 made in the determination of one of the component quantities,

and E_2, - - - E_n, be the errors due to other factors entering into the final result, then the total error of this result is, in the most unfavorable case, $E_1 + E_2 + \quad - \quad - \quad - \quad E_n$.

It is very instructive to calculate roughly, before beginning a given experiment, the influence which a small error in the evaluation of each factor will exert upon the accuracy of the final result. Such calculation will show the student upon which steps the greatest care should be expended, and also at what points an attempt at extreme accuracy means simply a waste of time and energy.

From a consideration of the applications of the above formula valuable suggestions as to methods of measurement are often obtained, whereby the percentage of error may be much reduced. Thus let it be required to determine the conditions most conducive to accuracy in the measurement of an electrical resistance by the slide wire bridge. The expression for the resistance measured by means of a slide wire bridge is

$$r = R \frac{a}{c - a} \tag{7}$$

where R is the known resistance in ohms, a the reading on the bridge wire, c scale divisions in length. In this case the expression for E becomes

$$E = e \frac{dr}{da} = e R \frac{c}{(c - a)^2} \tag{8}$$

and the relative error

$$\frac{E}{X} = \frac{E}{r} = e \frac{c}{a (c - a)}. \tag{9}$$

This expression will be a minimum for a maximum value of the denominator, $a (c - a)$. But since $a + (c - a) = c$ is a constant, $a (c - a)$ is a maximum when

$$a = c - a, \text{ or } 2a = c;$$

that is, the adjustment should be such as to bring the reading to the middle of the scale.

8. **Interpolation.** It is frequently desirable to evaluate a physical quantity beyond the limit of the subdivisions of the instrument at our disposal. Thus let it be required to weigh a body to 0.1 mg, while the smallest weight in the box of weights is 1 mg; or let it be required to determine the resistance of a piece of wire to 0.1 ohm, by the method of substitution when the known resistance is subdivided to ohms only. In such cases the method of interpolation is applied. Thus let x_0 be the observed quantity corresponding to the unknown quantity y_0. We are to select two quantities X and x, in the neighborhood of x_0 such that $X > x_0 > x$. Let the corresponding values of y_0 be Y and y. Then if these values lie near enough together so that we can assume that $Y - y$ is proportional to $X - x$, we find

$$y_0 = y + (x_0 - x)\frac{Y - y}{X - x}. \qquad (10)$$

9. **Hints on Computation.** The following suggestions regarding the computation of results will be found useful.

(*a*) *Taking the Mean.* In taking the mean of a set of readings, it is necessary to average only those parts of the various readings which **differ among themselves.** Thus if ten readings have the first three figures 264, and **differ** only **in the tenths** it is clearly unnecessary to average more than the tenths.

(*b*) *Significant Figures.* The student should avoid carrying a result out to a large number of decimal places, far beyond the point where the figures have any significance whatever. Thus if six readings of the barometer be 743.3, 743.2, 743.3, 743.1, 743.2, 743.3 mm., the mean is 743.233 mm., of which but *four* figures are significant and the tenths are uncertain since they cannot be read every time alike. If this reading be corrected for temperature the result should likewise be given to **tenths of a millimeter but no farther,** since nothing is known beyond that. In general the result should be carried to so many figures that the last, owing to errors, makes no pretension to accuracy, while the next to the last may be regarded as reasonably accurate.

(*c*) *Approximations.* In many cases where it is necessary to compute results from values, some of which are very small in

comparison to others, the labor may be greatly reduced by approximation formulae. Some of the more useful are given below, where a, b, c and d are to be regarded as very small in comparison to unity.

$$(1 \pm d)^m = 1 \pm m\,d$$

$$\sqrt{1 \pm d} = 1 \pm \tfrac{1}{2}\,d$$

$$\frac{1}{\sqrt{1 \pm d}} = 1 \mp \tfrac{1}{2}\,d \qquad (11)$$

$$(1 \pm a)\ (1 \pm b)\ (1 \pm c) \ldots = 1 \pm a \pm b \pm c \ldots\ldots$$

(*d*) *Supplementary Tables.* At the end of the book will be found a series of tables of mathematical and physical constants. The student will find it of advantage to consult these freely in the course of his work. While the values of the physical constants contained in these tables have been selected from reliable sources, the student is warned against the error of assuming that a value obtained in the laboratory is necessarily wrong, because it differs slightly from that given in the table.

CHAPTER I.

FUNDAMENTAL MEASUREMENTS.

10. **Fundamental Magnitudes.** Most physical quantities may be expressed either directly or indirectly in terms of the **fundamental units of mass, length, and time.** The first problem of physical measurement therefore has to do with the methods for evaluating certain quantities in terms of these fundamental magnitudes. Closely related to such fundamental measurements are the methods employed for the **measurement of angles** and of the **pressure** exerted by the atmosphere. On account of the great importance of these quantities the methods for their determination are included here. The instruments and processes described in this chapter being essentially those employed in the exercises which follow, a few words may be devoted to these fundamental measurements.

LENGTH.

11. **Contact Meaurements.** Many instruments for measuring length with a greater or less degree of accuracy involve the necessity of bringing the **instrument** into **contact** with the **object** to be measured. This may be effected by bringing the object between two jaws, one of which is movable, or between a movable part of the instrument and a fixed plane of reference. In any case accuracy of measurement requires that the contact between the object and the instrument shall be exact, and that the distance between the points of contact shall represent the true length of the object to be measured.

Hence it is of importance to see that the contact points of such instruments are scrupulously clean and true, and that the ends of the body are accurately faced and free from adhering matter of any kind. Such measurements are termed **contact measurements.**

12. **Methods of Subdivision.** Instruments for refined measurements of length usually involve the principle of the **micrometer screw** or of the **vernier,** or a combination of the two. Prominent among such instruments may be mentioned the **micrometer gauge,** the **spherometer,** the **vernier caliper,** the **cathetometer,** the **micrometer cathetometer,** the **comparator,** and the **dividing engine.** Of these the micrometer gauge and the spherometer employ the principle of the micrometer screw, the vernier caliper and the ordinary cathetometer employ the principle of the vernier, while the micrometer cathetometer, the comparator and the dividing engine employ a combination of the two. It is not the intention to describe the working of each of these instruments in detail, but so to give fundamental principles upon which each instrument is based, as to enable the student to make the application for himself.

13. **The Micrometer Screw.** In the **micrometer screw** we have a screw of fine thread working in a nut and furnished with a graduated head divided into some convenient number of aliquot parts. A complete rotation of the head advances or withdraws the screw by an amount equal to the distance between its adjacent threads, that is by some fraction of a centimeter, or of an inch. This small length is further subdivided by means of the divisions on the graduated head, so that the **least count** of the instrument, that is, the least length directly measurable by it, is the distance between the threads, divided by the number of divisions on the head. Still finer readings may be made by estimating tenths of these divisions.

14. **Exercise 1. The Micrometer Gauge.** In the **micrometer gauge** the end of the screw works against a fixed jaw. The number of complete turns of the screw is read from the stem of the instrument and the fraction of a turn from the graduated head. In use the least count of the instrument is first determined and recorded. The end of the screw is next brought into

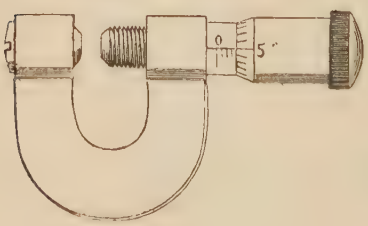

Fig. 1.

contact with the fixed jaw by slight pressure and the **zero reading** taken. The object to be measured is then brought between the jaws and the screw turned down until contact is made as before and the reading is again taken. The difference between this reading and the **zero reading** gives the thickness of the object, **expressed in the units marked upon the stem.** In determining the zero and final readings the mean of five readings is to be taken in each case. In more accurate instruments undue pressure upon the jaws is avoided by means of a ratchet head which slips as soon as contact is made. In using such instruments always turn slowly by means of this head and **stop as soon as the ratchet slips.**

FORM OF RECORD.

Exercise 1. The Micrometer Gauge.

Date.....
Object measured..... Pitch of screw.....
Micrometer gauge No..... Least count.....

Zero readings	Final readings
.....	.....
.....	.....
.....	.....
.....	.....
.....	.....
Mean	Mean .....

Thickness.....

15. **Exercise 2. The Spherometer.** In **the spherometer** the screw works in a nut supported by a frame having three legs of equal length, so placed that when the four points rest upon a plane the three feet form an equilateral triangle about the point of the screw at the center. The instrument is usually placed on a square of good plate glass. Notice of contact between the point of the screw and the plane is given by the instrument's **hobbling** or rocking on the plane. The screw is then carefully turned back until this hobbling just ceases. The zero reading is then taken. The object to be measured is then placed beneath the middle point and the screw turned down until contact is made, and the

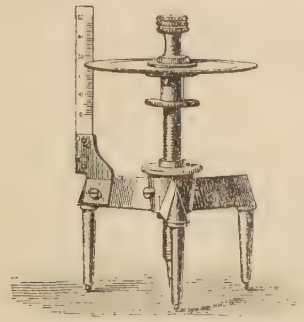

Fig. 2.

reading taken as before. The difference between the zero and final readings gives the thickness of the object.

In the more delicate form of the instrument, made by the Geneva Society, (Fig. 3), the screw point is connected by a system of light levers, to a **delicate pointer** which rises when contact is made. Readings are taken when the pointer rises to a fixed mark. In use avoid touching the graduated head with the fingers. Turn by means of the milled head provided for that purpose.

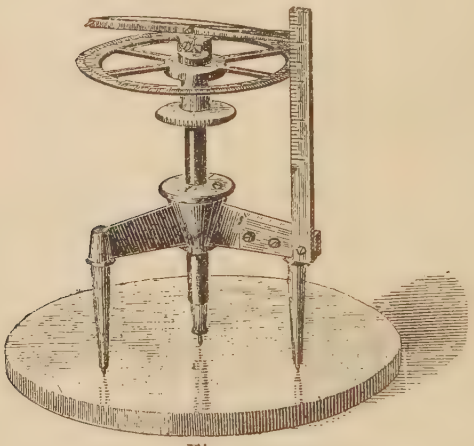

Fig. 3.

An extremely delicate means of determining the position of contact in the use of the spherometer, is by means of the **interference fringes** of sodium light. The spherometer is placed upon a piece of good plate glass and a small piece of glass with a good plane surface is placed under the central leg. When the surface of the glass is lighted by a sodium flame, the interference fringes appear at once. The slightest increase in pressure causes the lines to shift their position, thus indicating the position of contact.

FORM OF RECORD.

Exercise 2. The Spherometer. To measure the thickness of a piece of glass.

Pitch of screw..... Date .....
Spherometer No..... Least Count.....
Object measured.....

Zero readings	Final readings
.....	.....
.....	.....
.....	.....
.....	.....
.....	.....
Mean	Mean .....

Thickness of.....

16. **The Vernier.** The vernier is a device for subdividing the least division of a scale. It consists of a short subsidiary scale placed in front of, and in contact with the scale of the instrument, and is usually so divided that n divisions of the vernier correspond to $n-1$ divisions of the scale. The **least count**, *i. e.*, the least subdivision which may be read by the use of the vernier, is **$1/n$ of a scale division.** Thus, if S be the least division of the scale and V the least division of the vernier then

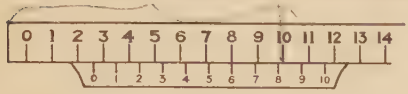

Fig. 4.

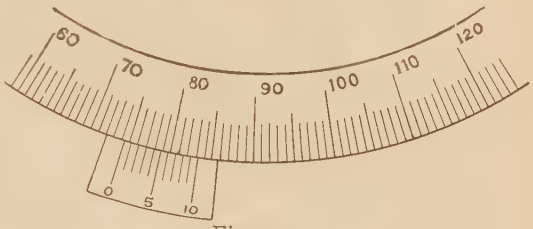

Fig. 5.

$$nV = (n-1)S$$

$$V = \frac{n-1}{n}S$$

or

$$\text{Least count: } S - V = \frac{1}{n}S \qquad (12)$$

In some cases n divisions on the vernier are made equal to one less than some multiple of n divisions of the scale; the formula then becomes

$$nV = (an-1)S$$

whence

$$\text{Least count: } aS - V = \frac{1}{n}S \qquad (13)$$

as before.

To read the vernier, first read the units of the scale **up to the zero of the vernier**; to this reading add **as many nths** of a scale division as are indicated by the vernier division which coincides with a scale division. Thus in Fig. 4 the reading is 2.6 scale divisions.

17. **Exercise 3. The Vernier.** Determine the least count of the verniers on five or more instruments.

FORM OF RECORD

Exercise 3. To determine the least count of the verniers on five different instruments assigned by the instructor.

Date.....

Name of Instrument	Value of S	n	$\frac{1}{n}S$

18. Exercise 4. The Vernier Caliper. The **vernier caliper** (Fig. 6), is an instrument in which the principle of the vernier is applied to the measurement of length. It consists of a pair of steel jaws, one of which is attached to the scale, the other to the vernier which slides upon the scale. In some instruments the movable jaw is pro- and slow motion screw for fine adjustment. Most instruments are adapted to inside measurements also, by means of a pair of rounded lugs attached to the ends of the jaws. When a separate scale is not provided for inside measurements, the thickness of these lugs must be added to the indicated reading.

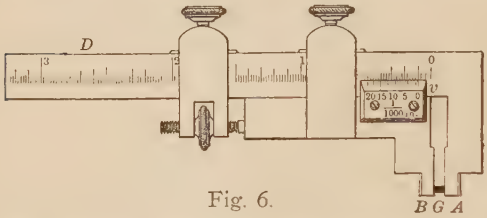

Fig. 6.

In use the value of a scale division S, and the least count, are first determined and recorded. The jaws are next brought together and the zero reading taken. The object to be measured is then placed between the jaws and the mean of several readings taken. The difference between the zero and final readings gives the length of the object.

FORM OF RECORD

Exercise 4. The Vernier Caliper. To measure the diameter and length of a brass cylinder.

Object measured..... Date .....
Vernier caliper No..... Least count .....

Zero readings	Diameter readings	Length readings
.....	.....	.....
.....	.....	.....
.....	.....	.....
.....	.....	.....
.....	.....	.....
Mean	Mean	Mean .....

Diameter ..... Length .....

19. **Line Measurements.** In the previous exercises on measurements of length it will be noted that in each case the measurement has been effected by **contact measurements**, that is, by bringing the measuring instruments into contact with the object to be measured. Frequently this is neither practicable nor desirable. In such cases **line measurements** are employed; that is, the distance between two lines drawn upon a body is determined, by focusing a microscope or telescope upon the lines in succession, and noting the readings of the vernier attached to the instrument. Line measurements are made by means of the cathetometer, the comparator and the dividing engine, and are used in all cases where great accuracy is desired.

20. **The Cathetometer.** **The cathetometer** is an instrument for measuring the difference in height between two points. It consists of a vertical standard, (Fig. 7), upon which slides a ring carrying a telescope and furnished with a vernier and a clamp for holding the telescope at any height desired. The instrument is provided with a level for bringing the axis of the telescope into a horizontal line, by means of adjusting screws. In most instruments the level may be rotated through a right angle about the standard as an axis, in order to secure the vertical position of the latter. The telescope is focused first upon one point and the reading taken, and then lowered by means of the clamp to the level of the second point, and set upon it, and the corresponding reading determined. For ease and accuracy in making the settings, the ring carrying the telescope is usually furnished with a slow motion screw. The difference between the two settings gives the vertical distance between the two points.

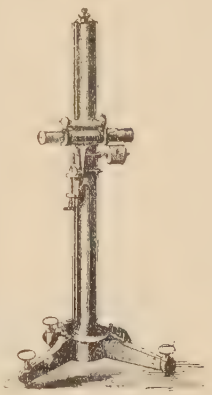

Fig. 7.

21. **Optical Micrometers.** **A micrometer** is a device for measuring minute lengths or changes in length with great accuracy. Such instruments are constantly employed in physical measurements and are made in a variety of forms. Since most

such instruments embody a combination of a micrometer screw and an optical system of some sort, it has seemed fitting to class them as **optical micrometers.**

(*a*) *Eye-piece micrometer.* Perhaps the simplest form of such micrometer consists of a finely divided scale, ruled upon glass and placed upon a diaphragm between the two lenses in the eye-piece of a microscope. The diaphragm must be so adjusted that the scale divisions are seen sharply defined on looking through the eye-piece turned toward a bright window. On introducing the eye-piece again into the microscope and focusing it upon some minute object, the image will be seen sharply outlined upon the ruled surface and the size of the image may be read off directly in terms of the units of the scale. By placing upon the stage of the microscope a **test plate,** or **object micrometer,** containing divisions of a known length, the **magnifying power** of the **instrument** and the corresponding **value** of a **single division** of the **eye-piece scale** may be directly determined. Once the value of a division is known the eye-piece may be used to measure lengths by simply counting the number of scale divisions covered by the magnified image.

Fig. 8.

(*b*) *Screw micrometer.* In the **screw micrometer** (Fig. 9), the tube of the microscope is moved in a line at right angles to its axis, by means of a **micrometer screw,** and the different parts of the image may be brought successively upon the cross hair of the microscope. The **advantage** of such an instrument is that its **range of measurement** may be **extended** to several centimeters.

When fitted to a microscope of low power and mounted upon a vertical standard the instrument forms a **micrometer cathetometer** (Fig. 10), and may be used to determine the distance between adjacent points with great accuracy. It may also be used to measure small changes in length as in the determination of **Young's modulus by stretching,** (Exercise 18).

(*c*) *Filar micrometer.* In **the filar micrometer** (Fig. 11),

the micrometer screw is used to displace across the field of view a glass plate carrying a pair of fine lines, or a light frame upon which are stretched a pair of **spider threads,** either parallel and close together or crossing at an acute angle. In most instruments there are also **two fixed threads** or hairs, at right angles to one

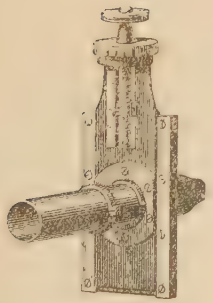

Fig. 9.

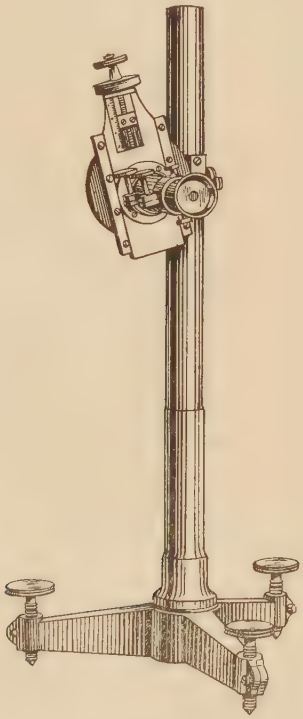

Fig. 10.

another. All the threads or cross hairs must **lie in the focal plane** of the **eye-lens** and be seen sharply defined when the micrometer is turned toward the bright sky.

When the micrometer is placed in the microscope or telescope and the instrument focused upon some object, the measurement of any feature of the image is effected by bringing the movable hairs **successively into coincidence** with the points under con-

sideration and noting the **corresponding readings** upon the **micrometer head.** In some instruments the count for the integral number of turns of the screw is shown in the field of view by means of a **toothed index,** where the moving hairs pass from one tooth to the next for one complete revolution of the head. Every fifth notch is made deeper than the others to assist in the reading. The zero may be taken at any definite point in the field as best suits the convenience of the observer. Usually the **divided head** is held in place by means of a **friction washer** or a lock nut which

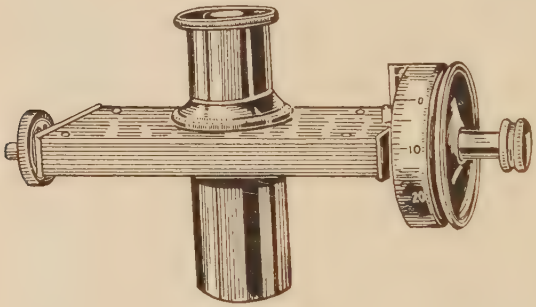

Fig. 11.

allows it to be adjusted for **any desired zero.** If there be a **fiducial mark** in the field of view to be used as a zero, then when the **parallel hairs include** this mark **symmetrically between** them the **divided head** should **read zero,** or be adjusted until it does.

The micrometer here described is used in the micrometer microscopes on dividing engines and comparators, in the reading microscopes of spectrometer and other finely divided circles, in telescopes on cathetometers and spectrometers, and on astronomical telescopes. The filar micrometer is also used for measurement of small angles, as shown in Article 33.

It is to be remembered that **lengths measured** by a micrometer placed in the eye-piece of a microscope are made upon the magnified image and are therefore **related directly** to the **magnifying power** of the instrument. Consequently in any series of measurements this **magnifying power** must be **kept constant,** *i. e.,* neither

the microscope objective nor the distance between the objective and the micrometer must be changed.

It is also to be observed that in making any setting the hairs must be brought up to the final position **from the same direction** in each case, in order to avoid the **lost motion** of the screw. If this position has been passed, the screw should be run back through at least two turns and brought up with greater care. Usually the hairs are carried from left to right across the field by direct pressure of the screw, and displaced in the opposite direction by means of a spring as released by the screw. Hence the best direction for approach to a setting is that in which the spring is compressed.

22. **The Dividing Engine.** In the **dividing engine** the micrometer screw is mounted in a rigid metallic bed so as to rotate freely, and carries upon it the movable nut which advances or recedes as the screw rotates. Attached to this nut is the traveling carriage upon which is mounted the device for ruling short lines transverse to the length of the screw. This ruling device may be operated either by hand or automatically by the machine itself, and is so arranged as to make **every fifth** and **tenth line distinctively longer** than the others, thereby facilitating the reading of the graduated divisions.

In modern machines the pitch of the screw is usually one millimeter and the divided head allows the screw to be advanced through any desired fraction of this length. Usually the smallest division of the head may be still further subdivided by a small vernier. In most machines the carriage is provided with one or more micrometer microscopes, whereby the operator is enabled to extend the graduation to lengths beyond the length of the screw.

In the machine shown in Fig. 12, the graduating device and the reading microscopes are fixed and the table carrying the surface to be graduated or the scale to be examined, moves with the motion of the screw. Otherwise the relations are the same as those described.

The dividing engine may also be used to measure lengths smaller than that of the screw with great accuracy, as soon as the pitch of the screw is known. To determine this, a standard scale

is supported horizontally beneath the reading microscope, and **accurately parallel** to the length of the screw. This adjustment is secured when the scale is seen in sharp focus throughout its entire length as the microscope is moved over it, and when the intersection of the cross hairs in the microscope cuts the divisions of

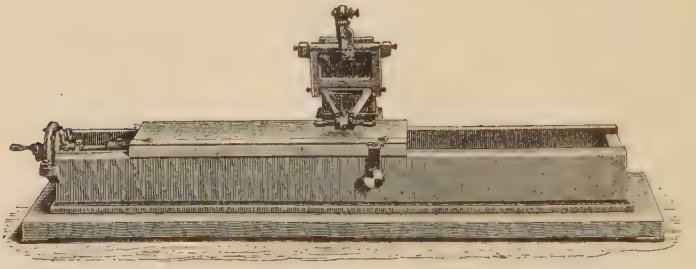

Fig. 12.

the scale at corresponding points throughout. The microscope is then set upon some **definite division** of the standard scale and the **number of turns** of the screw needed to carry the microscope to the next **similar division** is carefully determined. From the **value of the scale division** measured and the **observed rotation, the pitch of the screw** is readily **computed.**

The dividing engine is also useful for **measuring** accurately **small lengths,** for **calibrating** thermometer or other **capillary tubes,** for **study of the errors** of standard scales, for determining the constants and errors in the ruling of diffraction gratings, etc.

23. **Exercise 5. The Dividing Engine.** In order to measure with the dividing engine the distance between any two fixed points, the line connecting the points in question is placed accurately parallel to the screw of the instrument and readings taken first upon one point and then upon the other. The difference between the means of the two sets of readings gives the distance sought. In making any setting of the microscope care must of course be taken to avoid lost motion of the screw.

A smoothly planed, rectangular iron bar has ruled upon one of its sides two fine lines at right angles to its length. A third fine line ruled lengthwise of the bar, intersects the transverse lines normally, and enables the operator to set the bar accurately parallel to the length of the screw. The temperature of the bar is noted and at least five readings made upon each of the transverse marks. The observed distance is to be corrected for temperature by means of Table X.

FORM OF RECORD

Exercise 5. The Dividing Engine. To determine the distance between two marks on an iron bar.

Date.....

Temperature.....	Reading A.	Reading B.	Difference
	.....	.....	.....
	.....	.....	.....
	.....	.....	.....

Distance corrected to 20°C.

ANGLE.

24. **Measurement of Angle.** One of the most common measurements to be made in the physical laboratory is the determination of an angle. This may be the angle included between two lines on a plane surface, or that subtended by two points in space, or the angle swept out by a body as it rotates about an axis. For the determination of angular magnitudes many devices are employed of which only the more important will be described here.

25. **Definitions.** In **circular measure** an **angle** θ is defined as the **ratio** between its **subtending arc** s, and the **radius** r, or

$$\theta = \frac{s}{r}. \qquad (14)$$

The unit angle in circular measure is **the radian,** that is, an angle whose **subtending arc** is **equal** to the **radius.** Angles are also measured in degrees, minutes, and seconds (° ′ ″). The degree is the angle subtended by 1/360 part of a circumference.

From this it follows that the entire angle that may be described about a point is 2π radians, or 360°, and consequently

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ.2958$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$

In theoretical formulæ angles are usually expressed in radians, while measuring instruments give the values in degrees and fractions of a degree, hence frequent use must be made of the above equivalents in reducing from one system to the other.

Most measuring instruments are graduated in degrees and minutes and furnished with verniers or reading microscopes for subdividing the least division of the scale. The degree of accuracy required in the specific problem in hand must determine the refinements in measurement to be employed.

26. **Exercise 6. The Protractor.** In determining the angle included between two lines drawn upon a plane surface, as a sheet of paper or a blackboard, one of the simplest methods is to use a **protractor** (Fig. 13). This is a graduated circle with an arm rotating about its center. The center of the instrument is placed over the point of intersection of the two lines in question and the edge of the movable arm is brought into coincidence with the lines in turn and the corresponding readings of scale and vernier noted in each case. The difference between these readings is the angle sought. Measure the the angles of a triangle and check work by taking the sum. Measure each angle three times.

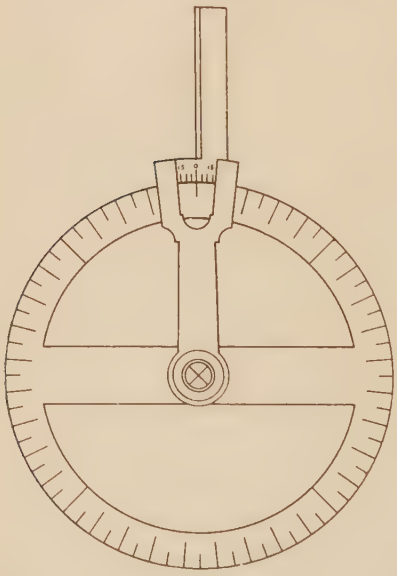

Fig. 13.

FORM OF RECORD

Exercise 6. The Protractor. To measure the angles of a triangle.

Date.....

Angle	*A*	*B*	*C*	
Mean				Sum =

27. **Exercise 7. Angles from Trigonometric Functions.** A very simple method for the approximate determination of an angle is by means of measurements which enable us to **compute** either its **sine** or **tangent,** preferably the latter. Thus if it be desired to know the angle ϕ subtended at the eye or at the objective of a telescope by a length l, placed symmetrically normal to the line of sight, at a distance L, a simple consideration shows that the angle is given by

$$\tan \frac{\phi}{2} = \frac{l}{2L} \qquad (15)$$

from which ϕ may be readily evaluated.

Determine, both in radians and in degrees, the angle subtended by some object in the laboratory at three different distances, and compare results thus obtained. To what degree of approximation may angles of 1°, 2°, 3°, 5°, and 10° be set equal to their tangents.

FORM OF RECORD

Exercise 7. To determine the angle subtended by an object at three different distances.

Object..... Date.....

	l	L	$\tan \phi/2$	ϕ	$\tan \phi$	Difference
1						
2						
3						

	1°	2°	3°	5°	10°
ϕ in degrees					
ϕ in radians					
$\tan \phi$					
Difference					

28. **Correction for Eccentricity.** Instruments intended for accurate work are provided with two or more verniers for the purpose of eliminating the error in reading the graduated circle due to eccentricity. This error is due to the fact that the center of the graduated circle and the center about which it or the vernier arms rotate, are never exactly coincident. This error is usually small, yet for instruments of precision it must be eliminated. This is readily accomplished by reading the circle at two points, 180° apart and combining the readings for every observation.

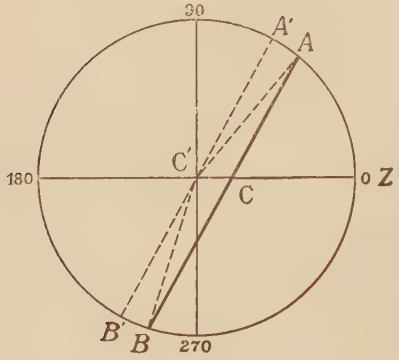

Fig. 14.

Thus in Fig. 14, let C be the true center of the graduated circle, and let C' be the center of rotation either for the circle or for the arm carrying the vernier. Then when the vernier stands at A, the axis of the telescope points in the direction of CA, while the reading at A corresponds to the direction $C'A$, and the error e, due to the displaced center is the angle CAC' *or* $A'C'A$. The true reading is the arc ZA' instead of ZA. On the other hand, the reading on vernier B, corresponds to a direction $C'B$, while the telescope looks in the direction CB, and the error e, is $BC'B'$ as before. In the first case the true reading R, is given by

$$R = A + e \tag{16}$$

and in the second by

$$R + 180° = B - e$$

whence

$$R = \frac{A + (B - 180°)}{2}, \tag{17}$$

the error due to eccentricity may be **completely eliminated** by taking the **mean of two readings** 180° apart. In practice it

is well to designate the right hand vernier as A and record the degrees as read from this vernier only, averaging the minutes and seconds as read from the two. It may also be shown that the error due to eccentricity may be eliminated by taking the mean reading of any number of equidistant verniers.

29. Telescope and Scale. In measuring small angular displacements as indicated by the tilting or rotating of a mirror about an axis the most common and efficient method is that of the **telescope and scale.** A telescope of low magnifying power (Fig. 15) is furnished with a scale, usually in millimeters, on which the figures are both inverted and perverted. This scale is so arranged that it may be held near the telescope and at right angles to its optical axis, in either a horizontal or vertical position. A small plane mirror m, whose angular displacement is to be determined, forms a virtual image of the scale which is viewed by the telescope. The scale is made as nearly parallel to the surface of the mirror as may be and the telescope adjusted until the scale division nearest the axis of the instrument is seen sharply defined on the cross hairs.

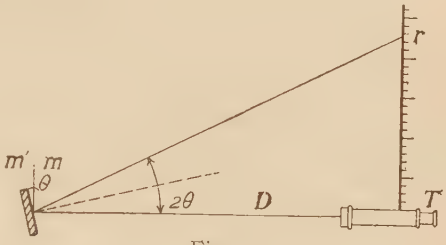

Fig. 15.

If now the mirror and its normal be rotated through an angle θ, then by the law for the reflection of light, the reading r, on the scale as seen in the telescope will form an angle of 2θ with the axis of the instrument. If D be the distance from the scale to the mirror then

$$\tan 2\theta = \frac{r}{D} \qquad (18)$$

from which the value of θ is readily determined. For accurate measurements the deflections are read first upon one side and

then upon the other, and the mean value for r used in computation. The equation (18) also shows that the sensitiveness of the method increases as the distance D is increased.

In many cases the ratio between the deflections is all that is needed. In such cases if two scale readings r_1 and r_2 have been observed we have

$$\frac{\tan 2\theta_1}{\tan 2\theta_2} = \frac{r_1}{r_2} \qquad (19)$$

or, since for small angles the tangents may be set equal to the angles themselves, we see that the **angular deflections** are **directly proportional** to the **scale readings.**

The method of telescope and scale is of universal application in work with **galvanometers, magnetometers, radiometers** and in all cases where slight angular displacements are to be determined. An exceedingly delicate instrument embodying the same principle is found in the optical lever.

30. **Exercise 8. The Optical Lever.** The **optical lever** is a device for measuring small lengths by means of the displacements of a beam of light reflected from a plane mirror. It consists of a stout bar (Fig. 16), supported upon four short, blunt pointed legs, of which one, at one end, is a micrometer screw, for purposes of adjustment and calibration. The bar carries at its middle point a plane mirror capable of rotation about a horizontal axis, and which may be clamped in any desired position.

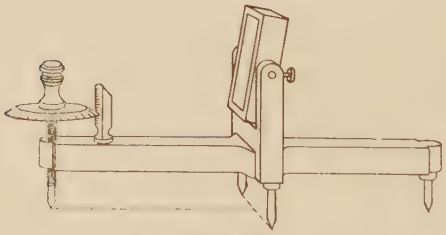

Fig. 16.

The instrument is placed upon a piece of good plate glass, in front of and some four or five meters distant from a reading telescope furnished with vertical scale. The telescope is focused upon the reflected image of the scale as seen in the mirror, and the

micrometer screw adjusted till all hobbling ceases. The mirror is then rotated until the central division of the scale falls upon the horizontal cross hair in the telescope. In making these adjustments care must be taken to free the plate and the points of the feet from all dust particles or lint, as otherwise the zero point will vary after the instrument has been tilted or moved on the plane.

After these adjustments have been made the object, as a small piece of microscope slide, is placed under the middle of the lever, the instrument is tilted so as to stand upon three legs, and the reading on the scale noted. The lever is then tilted in the opposite direction, placing a small weight if necessary upon the end previously elevated, and the scale reading again noted. The two positions of the lever are shown in Fig. 17. Then if the scale

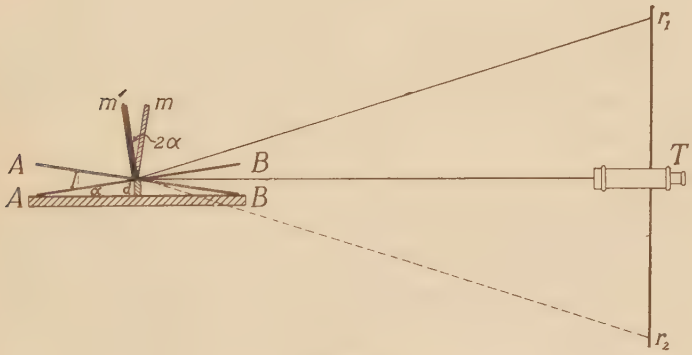

Fig. 17.

readings in the two tilted positions of the mirror be r_1 and r_2, if L be the distance from scale to mirror, l the half length of the lever and a the angle of tip in each case, we have for d, the thickness of the glass to be measured

$$d = l \sin a = l a \qquad (20)$$

since a is small.

Also since the total angular displacement of the mirror is $2\,\alpha$, the reflected beam is turned through $4\,\alpha$, and consequently

$$\frac{r_1 - r_2}{L} = 2\tan 2\alpha = 4\tan\alpha \qquad (21)$$

$$\alpha = \frac{r_1 - r_2}{4L} = \frac{d}{l} \qquad (22)$$

and

$$d = \frac{(r_1 - r_2)\,l}{4L} \qquad (23)$$

Make at least three determinations of r_1 and r_2 and take the mean. Determine l by means of the dividing engine.

FORM OF RECORD

Exercise 8. The Optical Lever. To measure the thickness of a piece of glass.

Date.....

Object measured.....

L	l	r_1	r_2	$r_1 - r_2 =$ $d =$
Mean				

31. **The Level Tester.** The **spirit level** is an instrument for testing a line or a surface as to its **deviation** from the **true horizontal.** It consists of a tube (Fig. 18), nearly filled with

Fig. 18.

ether or alcohol and closed at both ends. The inner surface of the upper side of the tube is accurately ground into a circular arc of large radius. When the tube is brought into the true horizontal

the enclosed air bubble, seeking the highest point of the arc, stands at the middle of the tube. Any change from the horizontal is indicated by the bubble moving toward the higher end. A scale of equal divisions is graduated upon the tube for reading the position of the bubble.

In accurate instruments the tube is held in its frame by adjusting screws so that its ends may be raised or lowered to bring the bubble into the middle when the level is placed upon a horizontal plane. By the horizontal screws the tube may be shifted laterally so as to bring its axis parallel to the line whose inclination is to be tested. In sensitive levels the vertical adjustment is seldom perfect, but it is more convenient to eliminate this error in use than to correct it by adjustment. This is done by reading the level in the **direct** and **reversed** positions each time and taking **half the difference between the readings** as the true position of the bubble.

32. **Exercise 9. Constants of the Level.** The value of **one division** of the level and the **radius of curvature of the tube** are termed the **constants of the level.** These may be determined by means of the **level tester** shown in Fig. 19. This

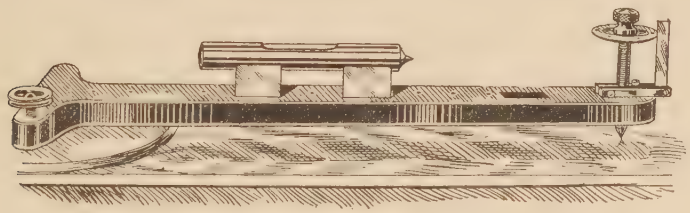

Fig. 19.

consists of a stout cast iron frame with a heavy horizontal metal bar hinged at one end so as to move in a vertical plane, and supported by a micrometer screw at the other end. By one turn of the screw the bar may be tilted through an angle ϕ whose value in **radians** is given by

$$\phi = \frac{p}{L} \qquad (24)$$

where p is the pitch of the screw and L the length of the bar.

The level is placed upon the bar and the readings on the micrometer and on the ends of the bubble recorded. The readings on the east and west ends of the bubble in the direct and reversed positions may be designated by e, w, e' and w' respectively. The screw is turned until the bubble moves over three or more divisions and the readings repeated. Proceed in this way by successive steps of from three to five divisions throughout the length of the graduation. Reverse the level and repeat the readings for the other half of the tube. From the data thus obtained compute the value of m, the distance the bar has been raised to move the bubble over one scale division. From the known values of L and m we may at once compute δ, the angular value of one scale division of the level, from the relation

$$\delta = \frac{m}{L} \tag{25}$$

where δ is expressed in **radians per division.** Reduce this value to seconds per division.

If there be n scale divisions graduated on the tube, then $n\,\delta$ is the angle subtended by this part of the tube at the center of the circle of radius R, of which it forms a short arc. If the length of the scale be l, then from the definition of an angle in **circular measure** we have

$$n\,\delta = \frac{l}{R} \tag{26}$$

$$R = \frac{l}{n\,\delta} = \frac{L\,l}{n\,m} \tag{27}$$

FORM OF RECORD.

Exercise 9. The Level Tester. To determine the value of a scale division, and the radius of curvature of a spirit level.

Date.....

$L =$ ..... $l =$ .....

Level direct				Level reversed			
Bubble		Screw		Bubble		Screw	
e	w			e'	w'		

Mean value of $m =$

$\delta =$

$R =$

33. **Small Angles by the Filar Micrometer.** Small angles may be measured with great accuracy by means of a telescope of large focal length in which the ordinary eye-piece has been replaced by a **filar micrometer.**

For laboratory experiments the telescope should have a focal length of at least 50 centimeters and should also have a slow motion in azimuth. A large **reading telescope** will do very well. The angle to be measured may be that subtended by two divisions of a brilliantly lighted scale or better by two **small pin holes** in a **piece of tin foil** placed before a small lamp. After the telescope has been sharply focused the micrometer is revolved in the tube until the **transverse hair connects the two points** in question, in order that the distance measured may be the **actual distance** between them. By the slow motion screw of the telescope the fixed hair is brought upon the image of one of the points. The **micrometer head** is then turned till the **movable hair coincides** with the **fixed hair** and the reading noted. The movable hair is then brought upon the other image and the reading upon the head again recorded, care having been taken to avoid lost motion of the screw in making the settings in each case.

The difference between the two readings gives the **angle** in terms of the **divisions** of the **micrometer head** and the **focal length** of the telescope **objective.** Let m be the measured distance between the images and F the focal length of the objective, both expressed in millimeters; then ϕ, the angle sought, is expressed in **radians** by

$$\phi = \frac{m}{F}. \qquad (28)$$

If r is the least reading of the micrometer, then ϕ', the least angle that can be measured is

$$\phi' = \frac{r}{F}, \qquad (29)$$

from which it is clear that the **delicacy** of measurement **increases** as the **focal length** of the **objective** increases.

34. **Exercise 10. Angular Measurement by the Filar Micrometer.** Take a piece of stiff, dark blue card board, 35 cms square, and cut from its center a hole 5 cms in diameter. Over this paste a piece of rather heavy tin foil in which have been pierced two small holes not more than 5 mms apart. Place the card board in front of a fishtail gas burner at a distance L, of at least 20 meters from the telescope. The bright yellow images are seen sharply defined upon the deep blue field. Repeat the measurements at least five times, setting the fixed hair first upon one image and then upon the other. Return the final value of the angle in seconds. Measure L and compare the measured value with the computed value of the angle subtended by the holes at the center of the objective. Compute also the least angle measurable with the apparatus.

FORM OF RECORD

Exercise 10. The Filar Micrometer. To measure the angle subtended by two points distant.....meters from the telescope.

Date.....

	Readings		
$L=$	1st Image	2nd Image	Difference
$l=$			
$r=$			
$F=$			
$\phi'=$			

M Mean value of m

ϕ Observed	ϕ Computed	Difference

35. **The Sextant.** The sextant is an instrument for measuring the angle subtended by two distant points or the angular elevation of a point above the horizon. It consists of a light frame (Fig. 20), carrying a telescope EF, a graduated arc AC, to the plane of which the axis of the telescope is parallel, and two

mirrors *I* **and** *H*, whose planes are normal to the plane of the arc. The mirror *I*, or **index glass**, is attached to the **index arm** *IB*, which rotates about *I* as a center and carries a **vernier** along the arc *AC*. The **horizon glass** *H*, is fixed in position and is silvered on its **lower half and transparent** on its upper half, so that on looking through the telescope, objects are seen through the upper half in the direction *EP*, while in the lower half are seen by reflection objects in the direction of *EP′*.

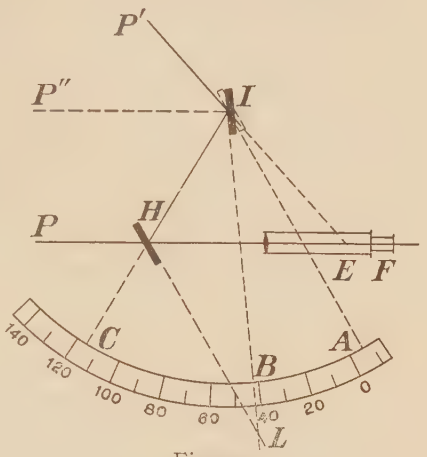

Fig. 20.

Let it be required to measure the angle *PEP′* subtended at *E* by the points *P* and *P′*. The telescope is pointed toward *P* and sharply focused. If the planes of the two mirrors are parallel the two lines of sight are parallel, and the two images of the object at *P* are coincident. If now the instrument be rotated about the line *EP* as an axis until the plane of the graduated arc passes through *P′*, and the index arm be then moved so as to bring the image of *P′* also into the field of the telescope, the two images may be made to coincide. In this case the light from *P′* has been reflected from the two mirrors *I* and *H*, and consequently it has been deviated through an angle equal to twice the angle between the mirrors.[1] The half degree marks on the arc are numbered as whole degrees, and hence the reading of the index gives directly the angle *PEP′*.

Figure 21 shows the usual form of the instrument, in which the various parts will be recognized without difficulty. The vernier arm is furnished with **clamp** and **slow motion screw** for making the final settings. It is important for accurate work that the light from the two objects should be as nearly as possible of the same intensity. To this end shades of colored glass are

[1] *College Physics*, Article 442.

attached to the instrument which may be rotated in front of either mirror at will.

Adjustments of the Sextant. In order that its readings may be reliable the sextant must be accurately adjusted and carefully handled. The instrument should fulfill the following conditions:

(*a*) *The plane of the index glass should be normal to the plane of the graduated arc.* This adjustment is secured by placing the eye near the index glass and adjusting the glass until the graduated arc and its reflected image appear to be in the same plane.

(*b*) *The horizon glass should also stand normal to the plane of the graduated arc.* This adjustment is attained when the two

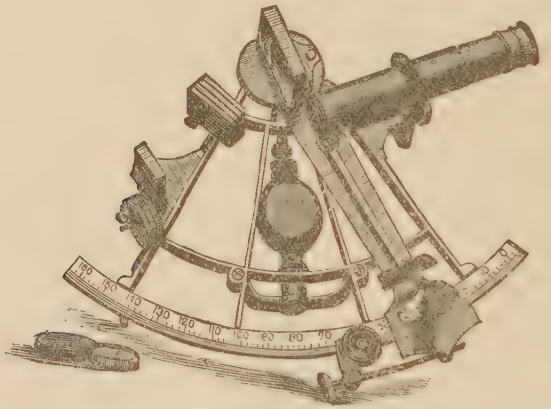

Fig. 21.

images of a distant object can be made to coincide exactly.

(*c*) *The axis of the telescope should be parallel to the plane of the graduated arc.* This adjustment is correct when the images of two stars, about 120° apart may be made to coincide exactly on either of two cross hairs, equidistant from the field of view.

The foregoing adjustments, when once carefully made should not require further attention. The instrument will be adjusted by the instructor in charge and beginners are cautioned against attempting such adjustments for themselves.

(*d*) *The index arm should read zero when the two mirrors are parallel.* This adjustment is rarely perfect, and when made

is subject to frequent changes. It is therefore better to correct for the error, thus introduced, and allow for it in all readings. It is also necessary to determine this correction every time the sextant is used as it varies from day to day.

(*c*) *Index correction.* The correction for the index error is determined by focusing the telescope upon some distant point and bringing the two images into exact coincidence. In this position the mirrors are parallel and the vernier should read zero. If it does not, suppose the reading to be A; then for an angle which gives the scale reading B, the true value is $B—A$. In some cases the value of A is negative, or the arm must be moved back beyond the zero in order to bring the images into coincidence. For this reason the graduation is continued slightly beyond the zero, as shown in Fig. 21.

36. Exercise 11. **Measurement of Angles by the Sextant.**

(*a*) Measure the angle subtended by two distant points.

(*b*) Measure the elevation of a point using an artificial horizon.

FORM OF RECORD

Exercise 11. To measure the angle between two points by the sextant.

Date.....

Index reading	Angle
Mean	.....
Index error.....	True angle .....

MASS

37. **The Balance.** Strictly speaking all determinations of mass are indirect. The balance is an instrument for the comparison of masses. In its simplest form it consists of a light beam turning readily about its middle point and carrying at its ends two scale-pans of equal weight. When disturbed the system oscillates about its position of equilibrium to which it finally returns. When masses are placed in the pans, it is evident that the original position of equilibrium will be resumed, only when the moments of the forces due to the action of gravity upon these

masses are equal. If now, we assume the arms of the balance to be equal, we may set the masses equal to each other when their moments have been shown to be equal; *i. e.*, when the balance resumes its original position of equilibrium.

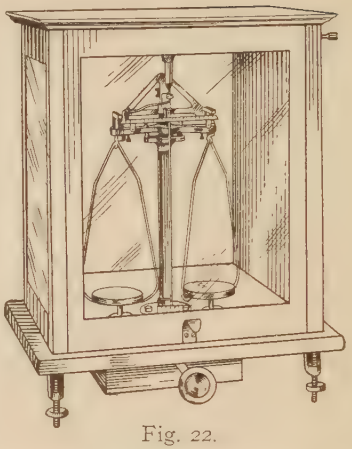

Fig. 22.

In balances of precision (Fig. 22), the beam and scale pans are hung from accurately ground knife-edges resting upon agate plates. When not in use, the knife-edges are relieved from the weight of the pans and beam by means of an arresting device. **This must always be used when weights are to be changed, or articles to be weighed are to be placed upon the scale pans or removed from them.**

The oscillations of the balance are observed by means of a long slender pointer moving in front of a graduated scale. Care should be taken to raise the system from the knife-edges, only when the pointer is at the middle of the scale. A system of light levers is usually placed under the pans, to maintain the balance in equilibrium for small differences of weight in the pans, and to prevent undue movements of the beam during rough weighing. The beam is generally divided from the middle outward, into n equal divisions, the last one coinciding with the knife-edge over the pan. By sliding upon this beam a small wire **rider** weighing n milligrams, weighings may be made directly to **milligrams.** For subdivision of the milligram the **method of oscillations** is used.

38. **Determination of the Resting Point.** Owing to the loss of time incident upon waiting for the balance to come to rest, it is usual to determine the final **position of the pointer** from a series of its successive turning points. Since the vibrations of the system are more or less damped, it is necessary to take the **initial** and **final readings** of the pointer on the **same side.** If we call

the central division of the scale zero, and readings to the right and left respectively plus and minus, then the resting point is found by averaging the means found for each side separately. For example, if the readings are

Left	Right
	+ 10.4
− 9.2	
	+ 10.0
− 8.9	
	+ 9.7
Mean − 9.05	Mean + 10.03

$$\text{Resting point} = \frac{-9.05 + 10.03}{2} = +0.49$$

This means that the pointer would finally come to rest at a point about 0.5 of a division to the right of the zero. The **resting point** should be determined both before and after making a weighing, and should remain constant if the balance be properly adjusted.

In some balances the scale divisions are numbered continuously from left to right. In the use of such instruments the readings are taken directly and the positive and negative signs are avoided, a method which seems more convenient than the former. If the scale divisions are not numbered, call the middle mark **ten.**

39. **Sensibility of the Balance.** If the rider be now placed upon the first division of the beam and the resting point determined as before, then the difference between these two resting points is the number of scale divisions through which the beam has turned for a difference in weight of one milligram. This is by definition the **sensibility** of the balance, and is usually termed the **sensibilty for zero load in the pans.** Since the sensibility varies with the load, it is always necessary to determine it for the specific load upon the pans.

40. **Exercise 12. To Make a Single Weighing.** A single weighing will not afford an accurate determination of mass, since the equality of the lengths of the arms is tacitly assumed, and this method, though commonly used, should not be employed in measurements of precision. Moreover it does not require a smaller number of observations than the much more precise method of **double weighing** described below. However, where only relative

weights are required, as in determinations of specific gravity, or in chemical analysis, single weighing is permissible, and is therefore described here.

First determine the **resting point** for **zero load.** Next place the object to be weighed upon the left hand scale pan and an estimated equivalent weight upon the right hand pan. Release the arrest very slightly and note the indication of the pointer. If the weight be too small it should be sufficiently increased to turn the pointer to the opposite side on the next trial. In this way the true weight may be rapidly approximated to the nearest gram, then to the nearest centigram. After this the **balance case should be closed,** the rider applied and the pan arrests turned down. Having found the weight to the nearest milligram, the balance is set swinging and the resting point is determined. The rider is then shifted **one whole division on the beam,** so as to bring the resting point on the **other side** of the zero position, and the resting point again determined. If now we call the three resting points p_0, p, and P, where P corresponds to the weight **greater** than the true weight, then dW, the fraction of a milligram to be added to smaller weight W, is

$$\frac{p - p_0}{p - P} = \frac{p - p_0}{s} \qquad (30)$$

where s is the sensibility for the given load. The true weight is therefore

$$W + \frac{p - p_0}{s} \qquad (31)$$

Thus suppose

$$p_0 = +0.49$$
$$p = +0.77$$
$$P = -0.15$$

then

$$\frac{p - p_0}{p - P} = \frac{0.28}{0.92} = 0.3 \text{ milligram.}$$

Care should be taken to avoid error in counting up the weights upon the scale pan. An excellent method is to write down the weights from the vacant spaces in the box and then check each weight as it is returned to the box.

FORM OF RECORD.

Exercise 12. To make a single weighing.

Balance No..... Date.....
Object to be weighed.....
Approximate weight .....

p	P	p_0
.....	.....	.....
....	....	....
....	....	....
$p =$	$P =$	$p_0 =$

$p - p_0 =$ .....; $s = p - P =$ .....

Weight of ..... $= W + \frac{p - p_0}{s} =$ .....

41. **Reduction to Weight in Vacuum.** The **weight** of a body **in vacuum** is[1]

$$W = w\left(1 + \frac{a}{d} - \frac{a}{D}\right) \qquad (32)$$

where w is the observed weight of the body, d its density, and a and D the densities of the air and of the weights respectively. In weighing a quantity of water with brass weights, this correction amounts to about 1.06 milligram, for every gram. It is therefore imperative, in the case of calibration of glassware by means of water or in similar problems to make the reduction to vacuum.

42. **Exercise 13. Double Weighing.**[2] In order to eliminate any inequality between the two arms of the balance the object may be weighed first in the left hand pan and then in the right. The true weight, W_0, is given by the equation

$$W_0 = \sqrt{W_1 W_2},$$

or, since for every good balance W_1 and W_2 are very nearly equal,

$$W_0 = \tfrac{1}{2}(W_1 + W_2). \qquad (33)$$

In practice the operations may be carried out as follows: Let p_0, p, and p' be the resting points for zero load, **load on the left**, and **load on the right** respectively; and let W be the **approximate weight**, *i. e.*, the weight sufficient to balance the load to within one milligram; then since

[1] *College Physics*, Article 76.

[2] For detailed treatment of the balance and its use, see Stewart and Gee, *Practical Physics*, Vol. I, pp. 63-94.

$$W_1 = W + \frac{p - p_0}{s} \text{ and } W_2 = W - \frac{p' - p_0}{s}$$

$$W_0 = W + \tfrac{1}{2}\left(\frac{p - p_0}{s} - \frac{p' - p_0}{s}\right)$$

$$= W + \tfrac{1}{2}\frac{p - p'}{s} = W + \Delta W. \qquad (34)$$

From this it is seen that in double weighing **the determination of p_0 is not required.** The **sensibility** may be determined with the load in either pan.

It will be instructive to determine W_1 and W_2 separately by **single weighing,** since the square root of their ratio will give us the ratio of the lengths of the arms of the balance. It should be noted that the method of double weighing, as described, requires no larger number of observations than the single weighing and is far more accurate.

FORM OF RECORD.

Exercise 13. To weigh a piece of brass, by the method of double weighing.

Balance No..... Date.....

Object to be weighed.....

Approximate weight, $W =$

Load left	Load right	1 mg. added
....	....	....
....	....	....
.....	.....	.....
$p =$	$p' =$	$P =$
$p - p' =$		$s = p - P$, (or $P - p'$) $=$
		$\Delta W = \frac{1}{2}\frac{p - p'}{s}$
		$\log 0.5 =$
		$\log (p - p') =$
		$\operatorname{colog} s =$
$\Delta W =$		$\log \Delta W =$
$W_0 = W + \Delta W$		

TIME.

43. **Period of Vibration.** In the case of a system vibrating freely about its position of equilibrium, the time elapsing between two successive passages of the system through the same point in its path in the same direction, is defined as its **period of vibration.** Most measurements of time in the physical laboratory consist in determining the period of vibration of some system, as a pendulum, magnetic needle, galvanometer needle, etc. The problem

presents itself most frequently as the determination of the **period of a pendulum**, and although what follows is applied directly to this end, the method is equally applicable to the case of any freely vibrating system.

44. **Method of Coincidences.** The process usually adopted is a modification of Borda's method of coincidences. The essence of the method consists in determining the time necessary for the pendulum to complete some large number of vibrations; from this, if the number of vibrations be known, the period of a single vibration is at once deduced. Perhaps the simplest way is to note the time occupied in counting 100 or 1000 vibrations. This method however, is both tedious and inaccurate, since owing to its monotony the observer is liable to make an error of one or even of ten vibrations in counting a large number. To this source of error is added the uncertainty of beginning and ending the count exactly upon the second.

In order to avoid the first source of error the **pendulum** is made to **keep count** of its **own vibrations** when compared with a clock beating seconds. The second error is minimized by attaching a pointer to the pendulum and observing its passage over a scale, or in work of greater accuracy, by viewing the pendulum through a telescope and noting the time of its passage over the cross-wires in the focal plane of the eye-piece. It is best to observe this passage when the pendulum is in the **middle of its swing**, and moving with its **maximum velocity.** The clock is connected electrically with a sounder and the beats are thus made audible throughout the room.

The pendulum to be timed is set swinging through a small arc not to exceed 3°. The observer at the telescope notes the transits of the pendulum image from left to right over the cross-wires and awaits the coincidence of such a transit with the click of the clock. He then notes carefully the number of seconds elapsing before the next passage of the image over the cross-wires **in the same direction** and estimates, as well as possible, the fraction of a second in tenths, thus gaining a roughly approximate period. Suppose the period is found to be somewhere between 2.3 and 2.5 seconds.

A coincidence is again observed and the seconds counted continuously until a number of fairly good coincidences have been observed. From this observation a closer approximation to the period can be obtained. Thus if the period is near 2.5 seconds, good coincidences will occur in 5, 10, and 15 seconds, *i. e.*, after 2, 4 and 6 complete vibrations of the pendulum. If on the other hand the period be nearly 2.3 seconds, then fairly good coincidences will be noted at 7 and 16 seconds, and a good one at 23 seconds, the intervals corresponding to 3, 7, and 10 vibrations. The imperfect coincidences at 7 and 16 seconds are due of course to the fact that the interval of 7 seconds is 0.1 second greater than the time needed for 3 swings, and that of 16 seconds is less by 0.1 second than that needed for 7 swings of the pendulum. In this way it is possible to determine the **provisional period** accurately to tenths of a second. The observer may distinguish between the **grade of coincidences,** by **underscoring** in his record, a moderately good coincidence with *one* line and an excellent one with *two.*

Several trials should be made and, if necessary, 50 or even 75 seconds counted, in order to determine this period with accuracy. Suppose it has been found to be 2.3 seconds. The observer again awaits a good coincidence, noting the seconds by calling each one *up to and including the second of coincidence,* **zero.** He then walks to the clock, counting the seconds as he goes, and on the tenth second reads the time, noting the seconds first, then the minutes and then the hour. This recorded coincidence is obviously ten seconds later than the observed one, but by counting *ten seconds* each time before reading the clock, the interval between the coincidences is preserved and no error is introduced.

A second coincidence is observed as soon as possible and recorded in the same way. Now the difference between the first and second readings of the clock gives the number of seconds corresponding to some **integral** number of swings of the pendulum, and a glance at the approximate period is usually sufficient to show what this number is. In case of doubt divide the time by the provisional period and take the nearest integer as the number of vibrations. (Why?) The number of seconds divided by the

number of vibrations, gives now the period to a closer degree of approximation than before. A third coincidence is observed and recorded as before. The interval **between this coincidence and the first** corresponds to a still larger number of integral swings of the pendulum. This larger number is found by dividing the seconds by the **period last deduced,** and the new value of the period is computed as before. Thus by successive observations we find intervals corresponding to a larger and larger number of vibrations, **using in each case the period last found.**

In this way the period of a pendulum may be readily and rapidly determined to thousandths of a second. After a little practice the student is able to judge a coincidence accurately to 0.1 of a second. Thus in the given example after 20 minutes of observation the pendulum would have made something over 500 vibrations, and the time needed for this number of vibrations would be determined to ± 0.2 of a second, or the period would be accurate to thousandths of a second.

Example. The following example will make the method and computation clear:

SIMPLE PENDULUM.

February 8, 1899.

Length, 130 cms.
Good coincidences, 7, 14, 16, 23.
Approximate period 2.3 seconds.

Coincidences 3 h. 19 min.	52 sec.	Interval in seconds	No. of Vibrations	Period in seconds
20	15	23	10	2.3
20	38	46	20	2.300
21	26	94	41	2.293
(21	50)	118	52	2.269 ??
22	30	158	69	2.289^{8}
23	57	245	107	2.289^{7}
24	29	277	121	2.289^{2}
30	19	627	274	2.288^{3}
30	51	659	288	2.288^{1}
33	22	810	354	2.288^{1}

It is to be observed that the period T gradually approaches a limiting value which becomes constant to thousandths of a second as soon as the number of observed vibrations, n, reaches a definite value. If the maximum error, e, made in taking any coincidence be ± 0.1 of a second, then the maximum error possible in any number of observed seconds is 0.2 of a second. Hence to have the period T constant to thousandths of a second, we must make

$0.2/n$ less than 0.001, or n must be greater than 200. Obviously n must be larger, the larger e becomes. How large must n be taken if $e = \pm 0.2$ seconds?

In case any observation gives a result **sharply at variance** with the others, the difficulty lies either in the arithmetical work, or in a false reading of the clock. The latter error renders the observation useless; it should be bracketed as indicated above, and the next taken with greater care. The **advantage of the method** is that **no single error** in reading the clock can **permanently vitiate** the result.

Instead of determining the number of vibrations by dividing the seconds by the last value of T deduced, it is much simpler to use the **preceding intervals** and **vibrations** as **measures** of the new. Thus the second interval 46, is manifestly double the first; hence the number of vibrations must be twice as many, or 20. In the third, the interval, 94 seconds, is twice the second + 2 seconds; the excess, 2 seconds, corresponds to an additional vibration; hence $n = 2 \times 20 + 1 = 41$. In the seventh determination the interval, 277 seconds, may be evaluated for n in a number of ways; for example, from the third we may have, after multiplying by three:

secs.	vibs.
282	= 123
277	
−5	= −2
277	= 121

or from the third and fifth thus:

94	= 41
158	= 69
252	= 110
277	
+25	= +11
277	= 121

For the measurement of small intervals of time the tuning fork furnishes an accurate and convenient method. For the practical application of this method see subsequent articles.

45. Exercise 14. Period of Torsional Pendulum.

FORM OF RECORD.

Exercise 14. To determine the period of a torsional pendulum, to 0.001 of a second.

Record as indicated above, p. 45.

46. Exercise 15. The Barometer. The **barometer**, (Fig. 23), consists of a closed tube of glass of uniform bore about 80 cms long, filled with mercury and inverted in a dish containing mercury. The free surface of the mercury in the vessel is in communication with the outer air. In the **cistern barometer**, the reservoir (Fig. 24), has a bottom of leather which is adjustable by means of a thumbscrew. A small **ivory point** extending downward from the upper surface of the reservoir, forms the **zero-point** from which the measurements indicated on the scale are reckoned. Before reading the barometer the mercury in the cistern must be so adjusted by means of the screw that the surface just touches the ivory point. The upper part of the tube is then gently tapped to free the mercury surface from the sides of the tube, and the vernier adjusted by means of the thumbscrew at the side, until, on looking through the slit in the barometer case, the upper part of the meniscus is seen to be just tangent to the line joining the sharp edges at the front and back of the vernier.

Fig. 24.

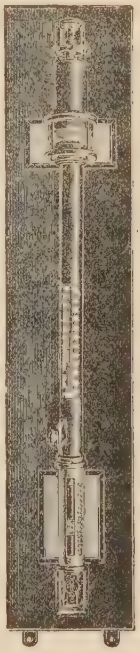

Fig. 23.

In the instrument made by Haak, of Jena, we have an **automatic adjustment** of the mercury in the cistern. The zero-point is the tip of a vertical tube connecting with a lower, auxiliary reservoir. Air is forced into the lower reservoir by means of a bulb, thus driving mercury into the reservoir proper and covering the zero point. On releasing the bulb the mercury flows out until the tip of the zero point tube is again exposed; the barometer is

then in adjustment for reading. The scale is etched directly upon the tube of the barometer, and fractions of a millimeter may be read with ease by means of the adjustable ring, at the top, which carries a fine line for subdividing the millimeter divisions.

Readings on the barometer must be corrected for temperature effects, and are reduced to 0°C by the use of the following formula:[1]

$$H_0 = H_t\,[1 - (a - b)\,t], \qquad (35)$$

where H_0 is the barometric height at 0°C; H_t is the barometric height at t°C; a is the coefficient of cubical expansion for mercury, ($a = 0.000181$ per degree), b is the coefficient of linear expansion for the material of the scale; (for glass, $b = 0.0000085$ per degree, for brass, $b = 0.000019$ per degree).

FORM OF RECORD.

Exercise 15. Adjust and read each barometer three times; correct for temperature and compare readings.

Barometer No. 1	Barometer No. 2	Date.....
Least count	Least count	
Readings	Readings	
.....	.....	
.....	.....	
.....	.....	
Mean	Mean	
Reduced to 0° C.....	Reduced to 0° C.....	

Express the barometric pressure in dynes per square centimeter.

[1] For reducing the barometric reading to standard conditions, viz.: 0° C, sea level, in latitude 45°, we have the complete formula

$$H_0 = H_t \frac{g}{g_{45}} \cdot \frac{1 + b\,t}{1 + a\,t} \qquad (36)$$

where g is the acceleration due to gravity at the place of observation and g_{45} 980.63 cm per sec. per sec.

CHAPTER II.

ELASTICITY.

47. **Definitions.** Elasticity is that property of matter by virtue of which a body resists the action of a force tending to change its shape or bulk, and resumes its original shape or bulk after the force is removed. If a body possess **elasticity of shape** it is called **a solid**; if it possess no elasticity of shape it is called **a fluid.**

Any change either of size or shape, produced by the action of a force upon an elastic body, is called **a strain,** and is measured by the **relative change** produced. The reaction against the deformation is called **a stress** and results in the appearance of a force resisting further deformation. After equilibrium ensues the applied and the resisting forces are equal. Hence stress may be evaluated in terms of the **applied force per unit area** of the cross-section upon which the force is exerted. In the metric system the unit of stress is the dyne per square centimeter.

Fluids possess **perfect elasticity of bulk,** *i. e.,* they return exactly to their former bulk on removal of the compressing force. Solids do not all recover their initial shape with equal promptness. In some cases the return is much retarded, especially after repeated or long continued distortion. This retardation is commonly termed **elastic after effect,** and is quite noticeable in metals. For every solid there is **a limiting distortion** beyond which the body, when freed from the distorting force, no longer completely regains its former shape. In engineering practice the **elastic limit** is usually measured in terms of the stress producing this limiting distortion.

48. **Hooke's Law.** When an elastic body is distorted within its limit of elasticity, the **opposing force** called out by the distortion, tending to restore the body to its original condition, is **proportional to the distortion.** This is known as **Hooke's Law,**

and as originally stated, "**ut tensio sic vis,**" expresses the proportionality between the distortion and the restoring force. The applications of this law are very numerous including every form of elastic reaction against strains produced by external agencies.

49. **Coefficients of Elasticity.** In general twenty-one coefficients would be needed to express completely the elastic nature of any solid. If however, the solid be isotropic, these twenty-one coefficients reduce to two: **the coefficient of volume elasticity,** e, and **the coefficient of rigidity,** n. The general expression for these coefficients is the quotient arising from **dividing the stress by the strain.**

50. **Coefficient of Volume Elasticity.** In the case of the coefficient of volume elasticity e, we have the **stress** measured by the applied pressure p, divided by the **compression produced,** where compression denotes the change in volume v, divided by the original volume V, or

$$e = \frac{p}{\frac{v}{V}} = \frac{p}{v} V. \tag{37}$$

In the case of a gas, the volume is at all times a function of the pressure to which it is subjected. Hence for gases it should be noted, that the coefficient of elasticity is to be defined in terms of the **change in pressure** and the corresponding **change in volume.**

Since these changes are conceived as being very small, if we assume a volume of gas V, to be subjected to a change in pressure dp, producing a corresponding change in volume dV, then for a gas,

$$e = V \frac{dp}{dV}. \tag{38}$$

It should be observed that the expression for the strain, dV/V, denotes a **dilatation if positive** and a **compression if negative.**

The coefficient c, however, has reference simply to the absolute value of the ratio Vdp/dV, and is therefore independent of the sign. The coefficient of elasticity of volume is the only one possessed by fluids, and is of special interest in all cases involving the propagation of disturbances through fluid media.

51. **Young's Modulus.** In solids in the form of wires or rods, subjected to longitudinal forces tending to produce either elongations or compressions, we are interested in the relative elongation or compression l, produced in length L, and cross section a, by a force of F dynes, when the body is free to contract or expand laterally. In general, longitudinal expansion is accompanied by lateral contraction and longitudinal compression by lateral distention. The measure or **modulus** of the elastic behavior of a solid under such conditions is known as **Young's Modulus**, and may be defined as the ratio between **longitudinal stress** F/a, and **longitudinal strain** l/L; that is,

$$M = \frac{F\,L}{a\,l} \tag{39}$$

or **Young's Modulus is numerically equal to that force which, when applied to a wire of unit cross section, would be sufficient to stretch it to double its length,** provided of course, that the elongation remained proportional to the force at all times.

52. **Simple Rigidity.** Besides the elasticity of volume, solids have, as we have seen, elasticity of shape as well. If a solid be so distorted that its shape alone is changed, it is said to have undergone **a shear.** Thus if we conceive all the particles in one plane in a body to be fixed, and all the remaining particles to move in planes parallel to this plane, and by amounts proportional to their distances from this plane, **such a distortion constitutes a shear.** The stress caused by such a shear is called **a shearing stress,** and the **coefficient of rigidity** or the **simple rigidity** is the quotient obtained by **dividing the shearing stress by the shearing strain.**

In order to learn how these quantities may be experimentally

determined, let us consider a circular cylinder (Fig. 25), of radius r, held vertically by a rigid clamp at the upper end and subjected to a torsional twist at the lower end. The effect of such a twist is to produce a shearing strain throughout the cylinder. Imagine the cylinder to be made up of a large number of tubes, one inside the other, and cut at right angles to the axis into a large number of circular sections. Each circular section would be composed of a large number of concentric rings. Now the shearing strain increases regularly from above downward from section to section, and when the lower end of the cylinder has been twisted through an angle θ, the **distortion** or **shear** for the **outer ring of the lower section will be the arc** $r\theta$, and the **shearing strain** is measured by the ratio of the shear to the distance of the sheared surface from the fixed end, or by $r\theta/L$; or in general, the shearing strain at any point in a cylinder is the circular displacement at that point divided by the distance from that point to the fixed end of the cylinder.

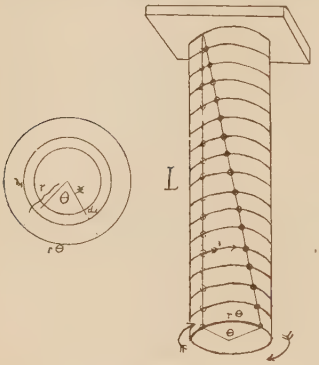

Fig. 25.

Again since the cylinder under stress is in equilibrium, the moment of the couple producing the shear must be balanced by the moment of the couple called out by the shear; or by Hooke's law, it must be proportional to the shear itself, hence also proportional to $r\theta/L$. Now by definition, the **simple rigidity** n, is the **proportionality factor** between **shearing stress** and **shearing strain**; hence we have the **shearing stress** $= nr\theta/L$.

Let us now consider the entire lower circular section of the cylinder, and in that section, a ring of radius x and of width dx. The shear will be $x\theta/L$, and the shearing stress will be $nx\theta/L$. The area of the elementary ring is $2\pi x\,dx$, hence the force, due to the shearing stress and equal to stress times area, is

$$dF = \frac{2\pi n\theta x^2\,dx}{L}$$

This force acting with a lever arm of x, gives an elementary mo-

ment of shearing force for the elementary ring of width dx, equal to

$$d\mathscr{T} = \frac{2\pi n \theta x^3 dx}{L} \tag{40}$$

For the entire section, the moment of the shearing force will be the sum of the elementary moments for all elementary rings, whose radii vary from zero, at the center, to r at the surface of the cylinder, and so the **moment of the torsional couple** called out by the shear, is found by integrating the expression in equation (40), or

$$\mathscr{T} = \int_0^r \frac{2\pi n \theta x^3 dx}{L} \tag{41}$$

$$= \frac{2\pi n \theta r^4}{4L}$$

or

$$\mathscr{T} = \frac{\pi n \theta r^4}{2L} \tag{42}$$

whence

$$n = \frac{2L\mathscr{T}}{\pi \theta r^4} \tag{43}$$

It must be observed that θ is here expressed in **radians.** If θ is measured in degrees, what correcting factor must be introduced?

In case of equilibrium $\mathscr{T}$ is equal to the moment of the torsional couple producing an angular twist θ, in a cylinder of radius r, and length L. By making the **angular twist equal to unity** we have the moment per unit angle

$$\mathscr{T}_1 = \frac{\pi n r^4}{2L} \tag{44}$$

and finally, by reducing the length L and the angular twist θ, each to unity we have

$$t = \frac{\pi n r^4}{2}. \tag{45}$$

The quantity t is called **the modulus of torsion; it is numerically equal to the moment of the couple required to produce unit angular twist in a wire of unit length.**

Summary:—We have now defined and derived expressions for the following quantities:

Coefficient of volume elasticity	$e = V \frac{dp}{dV}$	(38)
Young's modulus.....	$M = \frac{FL}{al}$	(39)
Torsional moment.....	$\mathcal{T} = \frac{\pi n \theta r^4}{2L}$	(42)
Simple rigidity.....	$n = \frac{2L\mathcal{T}}{\pi \theta r^4}$	(43)
Torsional moment per unit twist	$\mathcal{T}_1 = \frac{\pi n r^4}{2L}$	(44)
Modulus of torsion.....	$t = \frac{\pi n r^4}{2}$	(45)

In the experiments which follow several of the above quantities will be measured in one or more different ways.

53. Exercise 16. To Verify Boyle's Law. The apparatus consists of a cylindrical reservoir, (Fig. 26), formed of a glass tube some 25 cm long and 3 cm in diameter, into which is sealed a uniform tube B, some 30 cm long and 12 mm in diameter, closed at the top by a square-cut plug carefully cemented in, and at its lower end extending to the bottom of the larger tube. At the lower end of the reservoir is sealed on a second tube A, 5 mm in diameter and about 200 cm long. This tube is bent back upon itself about 10 cm below the reservoir, so as to be vertical and parallel to the tube B. It is terminated at its upper end by a small thistle bulb, for convenience in filling the reservoir with mercury. The instrument is mounted upon a suitable support carrying a scale graduated to millimeters at the side of the tube A, throughout its entire length. At the bottom the scale

stands between the two tubes so that readings upon the height of the mercury in the tubes *A* and *B*, may be made from opposite edges of the same scale. A small side tube is sealed to the reservoir near the top by means of which air may be forced into the reservoir from a small force pump. Before beginning the experiment the instrument is so adjusted that the shorter tube *B*, is about half filled with mercury when the air in the reservoir is at atmospheric pressure. Air is next driven in through the side at *C*, by means of the pump, until the mercury almost fills the long tube *A*. The air in *B* is now under a pressure measured by the barometer column plus the difference in height of the mercury in the tubes *A* and *B*, and is correspondingly compressed.

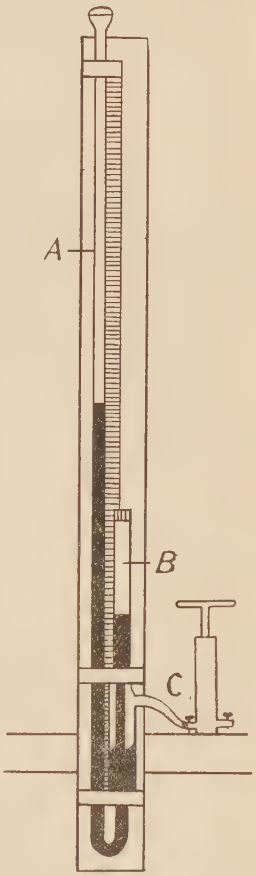

Fig. 26.

Varying pressures and corresponding volumes are successively secured by allowing small quantities of air to escape through the tube *C*. Readings are made upon the height of the mercury in *A* and *B*, and upon the lower end of the plug in *B*. Readings should be continued until the mercury in the tube *A* falls to the level of the mercury in the reservoir. The air must be allowed to come to the temperature of the room after each setting before the reading is taken. (Why?) If it be desired to take readings at pressures below that of the atmosphere, the pump is reversed and the air partially exhausted.

According to **Boyle's law,** the product of the pressure and the volume of gas is a constant, **for a constant temperature**[1], or

$$P.V = C. \qquad (46)$$

[1] *College Physics,* Article 77.

By definition, the coefficient of volume elasticity for a gas is

$$e = -V \frac{dP}{dV}.$$

Also from (46) by differentiation, we get

$$P\,dV + V\,dP = 0, \tag{47}$$

or

$$P = -V \frac{dP}{dV} = e. \tag{48}$$

Thus we see that for a perfect gas undergoing isothermal changes, the coefficient of volume elasticity e, is at all times equal to the pressure P. For the purpose of our experiment let a and b denote the observed heights of the mercury in the tubes A and B. Let P_0 represent the atmospheric pressure at the time of the experiment, $p = a - b$, the applied pressure, and let V represent a quantity proportional to the resulting volume of the air enclosed in the tube B. Then equation (46) becomes

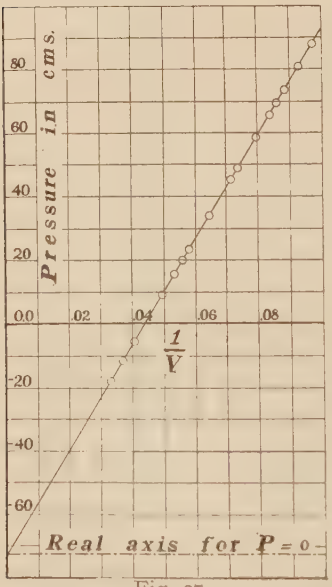

Fig. 27.

$$V(P_0 + p) = C \tag{49}$$

or

$$P_0 + p = C/V. \tag{50}$$

If now we plot the observed values of p on the Y axis and the reciprocals of the corresponding volumes on the X axis, (Fig. 27), our curve is represented by the equation

$$y = Cx - P_0, \tag{51}$$

the equation of a straight line cutting the Y axis at a point P_0 below the origin. This point may be considered the true origin, measured from which the values of y denote the successive values of e, corresponding to the related values of V. The tangent of the angle included between the curve and the X axis is equal to C. If all quantities be expressed in the

proper units, then C becomes *the gas constant*. Under what conditions does e approach zero?

FORM OF RECORD.

Exercise 16. To verify Boyle's law.

Reading on plug..... Barometer..... Date.....

Tube A	Tube B	$p = a - b$	V	colog V	$1/V$

Plot values of p and $1/V$.. Determine the value of P_0 from the curve and compare the result with the barometer reading at the time of the experiment.

54. Exercise 17. The Jolly Balance. In the Jolly balance the elastic body is a spiral spring mounted upon a suitable support and carrying at its lower end a pan to receive the substance under experiment. To this pan is attached a second pan to carry the substance when immersed in the water. The support consists of two telescoping nickel-plated tubes mounted upon an adjustable tripod base. The inner tube to which the spring is attached, is actuated by means of a rack and pinion A (Fig. 28), and is graduated in millimeters at its upper end through about 50 cms of its length. A vernier S, on the upper end of the outer tube, reading to tenths of a millimeter gives the elongation of the spring under any load when the **indicator** B attached to the lower end reads **zero.** This indicator consists of a small rod of aluminum furnished with two short cross arms, one at either end of a short vertical glass tube enclosing the rod and supported by an adjustable clamp. The glass tube is whitened at the back and at its middle has on the inside a fine black line passing entirely round it. The aluminum rod carries

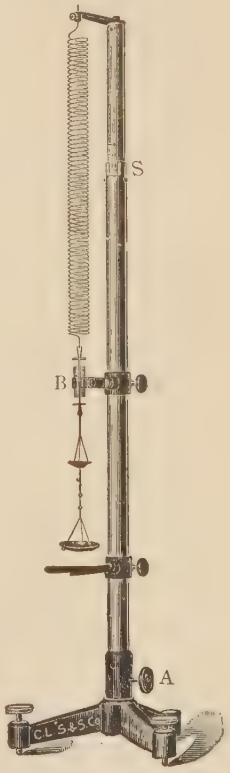

Fig. 28.

at its center a small cylinder bearing three equidistant black lines, the middle one of which is made to coincide with the line on the glass tube when the spring is brought to the **zero position.** An adjustable support carries a small vessel containing distilled water.

After the support is once adjusted it should remain in the same position throughout the experiment, since in this way the pan will always be immersed to the same depth in the water.

In use the support is adjusted and the reading r_0, taken with both pans empty. The substance under experiment is then placed in the upper pan, the support adjusted and a second reading r_1, is made. The substance is then transferred to the lower pan, the adjustment made and the reading r_2, taken. Then for the density of the substance we have

$$D = \frac{r_1 - r_0}{r_1 - r_2} d \qquad (52)$$

where d is the density of the water used.[1]

Derive and explain this formula.

For substances lighter than water a small piece of metal heavy enough to sink the substance, is placed in the lower pan and kept there during all three readings. The density is then computed as above.

For liquids the lower pan is removed and a suitable sinker, usually of glass is attached to the hook by a fine platinum wire. Readings are made with the sinker in the air, r_0, with the sinker immersed to a definite depth in the water r_1, and with the sinker immersed to the same depth in the liquid, r_2. Derive and explain the formula in this case.

It is necessary to exercise care in the use of the apparatus in order to secure trustworthy results. Air bubbles must be removed from the substance and from the lower pan before readings are taken. Determine the density of three solids: brass, zinc and paraffin, and of a solution of copper sulphate. Compare results with those obtained in preceding exercises.

[1] *College Physics*, Article 69.

FORM OF RECORD.

Exercise 17. Density by means of Jolly balance.

Density of..... Date.....

r_0	r_1	r_2	
.....	.....	.....	
.....	.....	.....	Density.....
.....	.....	.....	
Mean.....	.....	.....	

55. **Exercise 18. Young's Modulus by Stretching.** An iron bracket firmly attached to the wall near the ceiling supports a long brass wire which carries on its lower end, by means of a clamp, a cage for the reception of weights. Near the lower end of the wire is attached a needle upon the point of which is focused a micrometer cathetometer reading to 0.001 mm. In order to eliminate the yielding of the supporting bracket, a small rod hung from it is loosely attached to the wire, and bears on its lower end a small metal square or flag, near the needle point, so that both flag and point appear in the field at the same time and readings may be made upon them successively. An adjustable table is placed under the cage before weights are added and then gradually lowered, to avoid subjecting the wire to sudden jerks. A weight of two kilograms is left permanently upon the cage in order to free the wire from kinks and to insure uniformity of stretching.

Readings are taken upon the flag and the point, F_0, P_0, with only the zero load, two kilograms, on the cage, the mean of three readings being used in each case.

Let

$$F_0 - P_0 = l_0.$$

The table is then raised beneath the cage and two kilograms added. The table is then lowered and readings made as before.

$$F_1 - P_1 = l_1.$$

In this way readings are successively taken for weights of two, four and six kilograms in addition to the zero load. After this the weights are removed, two kilos at a time, readings being made as before until the zero load remains. From the values thus found

the stretch for two kilograms is computed for each reading. Thus from the reading with 6 kilograms added, we have

$$\frac{l_3 - l_0}{3} = l.$$

The mean of the values of l thus obtained, is taken as the stretch produced by two kilos. Having determined L and a, we have

$$M = \frac{F L}{a l} = \frac{2000 \cdot 980 \cdot L}{a l} \text{ dynes per cm.}^2$$

Determine the effect upon the final result, (a) of an error of 1 mm, made in the measurement of L; (b) of an error of 0.001 mm, made in the determination of each, l and r.

From a preliminary determination of these three quantities, compute the greatest error permissible in each one of them, if the result is to be accurate to within one per cent.

FORM OF RECORD.

Exercise 18. Young's modulus by stretching.

L..... Date.....

r..... $a = \pi r^2$.....

Weights	Flag	Point	l_n	l
			Mean.....	

Computation:

log 2000 = ...
log 980 = ...
log L = ...
colog πr^2 = ...
colog l = ...
log M = ...

M = ...

56. **Exercise 19. Verification of the Laws of Bending.** The bending of a beam supporting a weight is a function of the weight w, supported, of the three dimensions of the beam l, b, d, and proportional to a constant C, which depends upon the material of the beam and the manner in which it is supported. It is proposed in this exercise to determine experimentally these relations.

Let us assume for the purposes of the investigation, the general expression for the bending B,

$$B = C w^{\alpha} l^{\beta} b^{\gamma} d^{\epsilon}, \tag{53}$$

where α, β, γ, ϵ, are constant exponents to be determined by experiment. Since this expression is perfectly general it will include all possible bendings of the bar, obtained by varying the weight, length, breadth, or depth of the bar, either successively or simultaneously. In order to keep clearly in mind the relations under investigation it is best to vary but one of these quantities at a time, and observe the bendings produced under definite conditions. Thus with a bar of definite length, breadth and depth, **vary the load** successively and observe the bendings B_1, B_2, B_3, B_4, B_5, produced by the loads w_1, w_2, w_3, w_4 w_5.

Inserting related pairs of values of B and w, as B_1 and w_1, B_2 and w_2 in the general formula we have

$$B_1 = C w_1^{\alpha} l^{\beta} b^{\gamma} d^{\epsilon}$$

$$B_2 = C w_2^{\alpha} l^{\beta} b^{\gamma} d^{\epsilon}, \text{ and so forth.}$$

Passing to logarithms we have

$$\log B_1 = \log C + \alpha \log w_1 + \beta \log l + \gamma \log b + \epsilon \log d$$
$$\log B_2 = \log C + \alpha \log w_2 + \beta \log l + \gamma \log b + \epsilon \log d$$

$$\log B_1 - \log B_2 = \alpha (\log w_1 - \log w_2),$$

or

$$\alpha_{12} = \frac{\log B_1 - \log B_2}{\log w_1 - \log w_2} \tag{54}$$

Where α_{12} denotes that this particular value of α is derived from the related values of B_1, w_1 and B_2, w_2.

Again by using a constant difference in load, w, and **varying the lengths** l, we obtain a series of bendings B'_1, B'_2, B'_3, B'_4 B'_5, corresponding to the lengths l_1, l_2, l_3, l_4, l_5. Applying the

general formula to the values of the bendings B'_1 and B'_2, for lengths l_1 and l_2, we have, in the same way as before,

$$\log B'_1 = \log C + \alpha \log w + \beta \log l_1 + \gamma \log b + \epsilon \log d$$
$$\log B'_2 = \log C + \alpha \log \mathrm{w} + \beta \log l_2 + \gamma \log \mathrm{b} + \epsilon \log d$$

from which

$$\beta_{12} = \frac{\log B'_1 - \log B'_2}{\log l_1 - \log l_2} . \qquad (55)$$

Similarly by determining the bending for a definite weight in bars of the same metal, and having l and d the same, but **varying the breadth** b, we find for the successive breadths b_1, b_2, &c., the corresponding bendings B''_1, B''_2, &c., and deduce the relation

$$\gamma_{12} = \frac{\log B''_1 - \log B''_2}{\log b_1 - \log b_2} , \qquad (56)$$

and finally by **varying the depth,** we have

$$\epsilon_{12} = \frac{\log B'''_1 - \log B'''_2}{\log d_1 - \log d_2} . \qquad (57)$$

The apparatus (Fig 29), consists of a graduated scale upon which slide two knife-edge supports for the metal bar, one of which is connected to a battery. At the middle of the graduated scale is mounted a micrometer screw whose point rests upon the back of a knife-edged stirrup, sitting upon and at right angles to the bar at its middle point, and carrying the pan in which the weights are placed. The micrometer screw is connected to the other pole of the battery and the circuit is completed through the bar itself and the stirrup. A telephone receiver placed in the

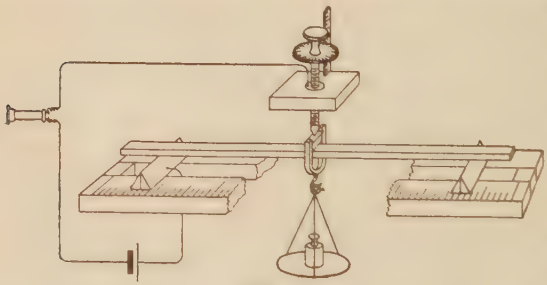

Fig. 29.

battery circuit gives notice of contact between the point of the screw and the stirrup.

In use the supports are placed at equal distances on each side of the micrometer screw, and the bar, with the stirrup on it, is so placed upon the supports as to leave equal lengths projecting over the supports at each end. The bar is brought directly under the screw point and the stirrup placed accurately at right angles to the bar and under the screw. The pan containing the weights must hang clear of all objects. The apparatus having been perfectly adjusted the length of the bar l, between the supports, is read and recorded. The micrometer screw is then slowly turned down until the telephone gives notice of contact between the screw and the stirrup. The screw is then carefully turned back until the snap due to breaking the circuit is heard in the telephone. The zero reading is then taken, using the mean of three in each case.

In place of the battery, telephone and micrometer screw, we may set one end of an optical lever upon the stirrup, the two middle legs resting upon a fixed support near the bar, and take readings by means of a telescope and vertical scale. The absolute value of the readings of the apparatus may be determined once for all by depressing the middle of the bar by means of a micrometer screw and noting the related scale readings.

For the purposes of this exercise however, all that is necessary is the relative depression for the different conditions, and these are given by the scale readings. This method has the combined advantages of greater speed and greater accuracy in making the readings.

For this experiment four bars of brass of rectangular cross-section and about 70 cms. in length are employed. One of the transverse dimensions is the same for all bars so that when placed with this dimension vertical, we have a series of bars of constant depth. The other transverse dimension, for the four bars, differs in each case, so that when this dimension is placed vertical we have a series of bars of constant breadth and variable depth.

To determine the bending due to 100 grams, or to any weight whatever, it is better to take the zero reading with an initial weight in the pan. Then to find the bending for any given load,

as 100 grams, the bending is taken for the initial load **plus the 100 grams,** and the difference between this reading and the initial reading gives **the bending for 100 grams.** A new initial load is taken and the same process is repeated, thus giving a second value for the bending for 100 grams; a third initial load is chosen and a third value for B is determined. The mean of the three values thus found is the bending for 100 grams. **This method is to be pursued in finding the bending for any weight whatever.**

FORM OF RECORD.

Bending for 100 grams Date.....
Bar No..... Length.....

Load	Readings	Means	Difference for 100 grams
	.....		
50 g.	.....	.....	
	.....		
			.....
	.....		
150 g.	.....	.....	
	.....		
	.....		
75 g.	.....	.....	
	.....		
			.....
	.....		
175 g.	.....	.....	
	.....		
100 g.		.....	
			.....
200 g.		.....	

Mean bending = B *for* 100 *g*

The exercise may now be completed under the following heads:

(*a*) *Vary* w *and determine* B *for five different loads.* Use as loads the weights 75, 100, 150, 200, and 250 grams. Use a single bar about 60 cms. in length. Do not at any time exceed 300 grams. Determine B_1, B_2, B_3, B_4, B_5, for w_1, w_2, w_3, w_4, w_5. Apply formula (54), combining the observations two by two. The mean of the ten values of a obtained in this way is taken as the value of a to be substituted in the general formula. In case any value of B as B_n, has been wrongly determined, then every value

of a containing the subscript n will differ sharply from the rest and will indicate that the nth observation should be taken with greater care. A glance at the tabulated values of a will generally reveal any such pronounced error of observation.

FORM OF RECORD.

Exercise 19. Verification of the laws of bending.

First part as given on preceding page. Date.....
Second part thus:

B	cms.	log B	w	grams	log w
B_1	.....	.....	w_1	.....	.....
B_2	.....	.....	w_2	.....	.....

and so on.

$\log B_1 - \log B_2 =$	$\log w_1 - \log w_2$	$a_{12} =$ .....
$\log B_1 - \log B_3 =$	$\log w_1 - \log w_3$	$a_{13} =$ .....

(*b*) *Vary l and determine B′ for a constant load $w = 100$ grams.* For lengths 30, 40, 45, 50, 55 cms, determine B'_1, B'_2, B'_3, B'_4, B'_5 for the constant load $w = 100$ grams. Apply formula (55), combining the observations as in (*a*). Take as the final value for β the mean of the ten values found as above. Record as in (*a*), substituting l for w.

(*c*) *Vary b and determine B″, for a constant load of 150 grams.* Use bars 1, 2, 3, 4, **with constant depth.** Measure b_1, b_2, b_3, b_4, by means of the micrometer gauge. Determine for b_1, b_2, b_3, b_4, the corresponding values B''_1, B''_2, B''_3, B''_4. Apply formula (56), computing as in (*a*) and (*b*). Record as before.

(*d*) *Vary d and determine B for a constant load, $w = 150$ grams.* Use bars **with constant breadth.** For depths d_1, d_2, d_3, d_4, determine the bending B_1''', B_2''' B_3''', B_4''. Compute from formula (57). Record as before.

(*e*) Insert final values of α, β, γ, ϵ, in the general formula. Formulate the laws of bending in words.

Curves: The observations may be graphically represented by curves. In cases of direct proportionality between the bending and the quantity which was varied in the experiment, plot this

quantity as abscissa and the corresponding bending as ordinate. The curve should be a straight line. If the exponent of the variable quantity is not unity a straight line may be obtained by plotting the logarithms. Plot the curves for each of the four series of observations.

57. Exercise 20. Young's Modulus by Flexure. As we have learned in the last exercise, the bending of a bar of length l, breadth b, and depth d, under a load w, is expressed by the equation

$$B = C \frac{w l^3}{b d^3} . \qquad (58)$$

It has already been observed that the constant C, depends upon the mode of support and the material of which the beam is composed. It is shown in the theory of elasticity that when the beam is supported by its ends the value of the constant relating to its mode of support, is 1/4, and when supported from one end its value is 4. There remains therefore, the undetermined part of our constant C, depending upon the material of the beam. It may be shown from mathematical analysis that this part of the constant is $1/M$, where M is Young's modulus for the material in question. If the bar be supported at the ends as usual, then we may write[1]

$$B = \frac{w l^3}{4 M b d^3}$$

or

$$M = \frac{w l^3}{4 b d^3 B} . \qquad (59)$$

If now we insert in this formula the dimensions of bar No. 1, we shall find for any bending B and the related load w, a value for M very nearly that obtained in Exercise 18 as Young's modulus for the material of the bars.

[1] It may be shown directly that the expression for M as given above, is true for all bars supported at the ends and loaded at the middle. Such proof would, however, exceed the limits set for this text. See Stewart and Gee, Vol. I, pp. 162-195.

Moreover it appears that Young's modulus is concerned here if we consider attentively what takes place in the bending of a bar. We see that upon the under side the bar is stretched, while upon the upper side a compression must ensue. Now the resistance to this stretching stress on the one side and to the compressing stress on the other, form the two parts of a couple tending to right the bar under its load, and the stress divided by the strain gives again Young's modulus for the material in question.

FORM OF RECORD.

Exercise 20...To determine Young's modulus of brass and steel by the method of flexure.

I. For brass.
From Exercise 19 (*a*)

$w =$	$\log w =$ .....
$l =$	$\log l^3 =$ .....
$b =$	$\operatorname{colog} 4 =$ .....
$d =$	$\operatorname{colog} b =$ .....
$B =$	$\operatorname{colog} d^3 =$ .....
	$\operatorname{colog} B =$ .....
$M =$	$\log M =$ .....

Date.....

II. For steel.
(a) First part as on page 64.
(b) Computation as above.

58. **Exercise 21. Simple Rigidity.** As shown in equation (43) the expression for the simple rigidity of a cylinder under torsional stress is

$$n = \frac{2 L \mathcal{T}}{\pi \theta r^4}.$$

where $\mathcal{T}$ is the moment of the torsional couple needed to produce an angular twist of θ radians in a circular cylinder of length L and radius r. In order to determine n we must measure the four quantities $\mathcal{T}$, θ, L, and r. This is most readily done by means of the following apparatus.

A metal rod (Fig. 30) some 150 to 200 cm in length, is supported horizontally and held firmly at one end by a clamp. The

other end is held clamped in the axis of a light aluminum wheel, some 20 cm in diameter, which is mounted on ball bearings, and graduated on its face through 180°. The wheel is furnished with a double vernier for reading its position at any time, and carries on its rim a flat steel tape to which is attached the holder for the weights producing the torsion.

The manipulation is as follows: A zero load of some 50 grams is first placed in the pan and the zero readings taken. A load of 100 grams is then added and the reading again taken. The differ-

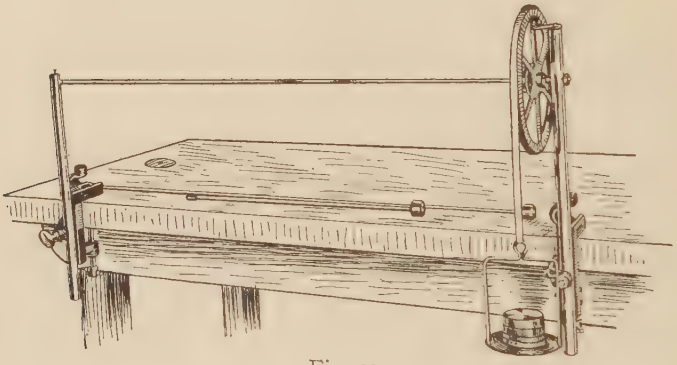

Fig. 30.

ence between the two readings is the angular twist for 100 grams. The load in the pan is again increased by 100 grams and the reading made. Half the difference between this reading and the first is likewise the twist for 100 grams. The load in the pan is increased in this way by successive steps of 100 grams until 500 grams have been added and then diminished by similar steps until the zero load remains. The twist for 100 grams is computed from each reading by combining it with the zero reading as shown above. The mean of the values thus found is the angular twist θ, expressed in degrees, for a load of 100 grams. The length of the rod l, and the radius of the wheel l, are to be determined by measurement. Determine the radius of the rod from five measurements of the diameter by means of the micrometer gauge.

FORM OF RECORD.

Exercise 21. To determine the simple rigidity of two metals.

Date.....

Load	Readings	Diff. for 100 gms.
50 gms.	.....	.....
150 "	.....	.....
250 "	.....	.....
350 "	.....	.....
		Mean
		.....

Computation:

$$n = \frac{360 \cdot L \cdot m \cdot g \cdot l}{\pi^2 \theta r^4}$$

$\log 360 = ...$

$L = ...$ $\log L = ...$

$m = 100$ $\log m = ...$

$g = 980$ $\log g = ...$

$l = ...$ $\log l = ...$

$\theta = ...$ $\operatorname{colog} \theta = ...$

$r = ...$ $\operatorname{colog} \pi^2 = ...$

$\operatorname{colog} r^4 = ...$

$\log n = ...$

$n = ...$

59. **Exercise 22. Simple Rigidity of a Brass Wire from Torsional Vibrations.** We have learned from Hooke's law that, within the limits of elasticity, the restoring force called out by any distortion is simply proportional to that distortion. An important consequence of this law is, that if a heavy body be suspended by a wire and the wire be twisted through a moderate angle and then released, the restoring force is continually proportional to the distortion. The motion induced is simple harmonic and consequently the vibrations of the body are isochronous. Now if I be the moment of inertia of the body, and $\mathcal{T}_1$ the moment of the torsional couple divided by the angular twist, then T, the time of a complete vibration of the system, is given by the equation

$$T = 2\pi \sqrt{\frac{I}{\mathcal{T}_1}} \tag{60}$$

from which

$$\mathcal{T}_1 = \frac{4\pi^2 I}{T^2} . \tag{61}$$

We have also seen from (44) that the coefficient of simple rigidity is defined by the equation

$$\mathcal{T}_1 = \frac{\pi n r^4}{2L}$$

whence, equating values for $\mathcal{T}_1$ and solving for n, we have

$$n = \frac{8\pi I L}{r^4 T^2} , \tag{62}$$

an expression involving only quantities amenable to measurement.

The apparatus consists of a heavy metal cylinder suspended by a brass wire and furnished with a light pointer moving over a horizontal circular scale. By means of an adjustable clamp the length of the torsional pendulum may be varied within wide limits. The times of vibration T_1, T_2, T_3, for lengths L_1, L_2, L_3, are determined to thousandths of a second by the method given in Article 44. The moment of inertia I, of the cylinder is computed from its mass M, and radius R. The radius of the wire is determined by means of the micrometer gauge. The insertion of any pair of related values of T and L in formula (62) gives a value for n. Compute the three values and return the mean as the value found for n.

FORM OF RECORD.

Exercise 22. Simple rigidity of brass wire from torsional vibrations.

Date.....

First part as given under Exercise 14

Second part thus:

L_1 ..... T_1 .....
L_2 ..... T_2 .....
L_3 ..... T_3 .....

$M =$ .....
$R =$ .....
$I =$ .....
$r =$ .....

Computation:

$$n = \frac{8 \pi I L}{r^4 T^2}$$

$\log 8 =$ .....
$\log \pi =$ .....
$\log I =$ .....
$\log L =$ .....
$\operatorname{colog} r^4 =$ .....
$\operatorname{colog} T^2 =$ .....

$\log n =$ .....
$n =$ .....

CHAPTER III.

PENDULUM EXPERIMENTS AND MOMENT OF INERTIA.

60. The Simple Pendulum. For practical purposes the pendulum may consist of a lead ball about 3 cm in diameter suspended by a fine steel wire some three meters in length. The wire is supported from the upper end of a substantial wooden bar two and a half meters in length about 10 cm wide and 5 cm thick, fastened firmly to the wall in a vertical position. The bar has let into its front face a brass strip ruled to millimeters and reading continuously throughout its length of 250 centimeters. Over this bar slides a wooden clamp which may be set at any position, and carries on its front a short, horizontal brass rod 10 cm long and 1 cm in diameter, furnished at its outer end with a diametral slit set vertically, and adjustable by means of a set screw. In this way the fine steel wire of the pendulum may be clamped in the slit of the rod at any point and the length of the pendulum varied between 50 and 300 centimeters.

Such a pendulum approximates very well an ideal simple pendulum. If h be the distance from the lower side of the clamp slit, to the center of the ball and r the radius of the ball then the length l, of the equivalent ideal simple pendulum is given by

$$l = h + \frac{2r^2}{5h} \qquad (63)$$

Compute the error produced by omitting the last term of this formula in the above pendulum, when h is 100 cm.

61. Exercise 23. Law of the Simple Pendulum. The object of this exercise is to investigate the relation between the period of a simple pendulum and its length. Since the most casual observation shows that the period of a pendulum is some function

of its length, we may assume as a general expression for the existing relation

$$T = C\, l^m \qquad (64)$$

where C and m are constants to be determined by experiment. Passing to logarithms and solving for m, as in Exercise 19, we find for m the value

$$m_{12} = \frac{\log T_2 - \log T_1}{\log l_2 - \log l_1} \qquad (65)$$

From a series of observations on five pendulums of different lengths we get, by combining as in Exercise 19, ten values of m. The mean value of m so determined, when inserted in the equations connecting related times and lengths, gives five independent equations for C, of the form

$$\log C = \log T - m \log l. \qquad (66)$$

The mean value of C thus determined, and the mean value of m when inserted in equation (64), give the relation sought. The exercise is to be completed as follows:

(*a*) Determine to thousandths of a second the period of vibration of a pendulum for five lengths 120, 140, 160, 180 and 200 cms.

(*b*) Compute from these observations the values of m and C as described above, observing the arrangement adopted on page (65).

(*c*) Plot curve, using values of log T as ordinates and those of log l as abscissae.

FORM OF RECORD.

Exercise 23. To determine the law of the simple pendulum.

Date.....

(*a*) Use form of record given under Art. 44.

(*b*) l	log l	T	log T	m	C

62. **Exercise 24. Computation of g.** From the well known formula for the time of vibration of a simple pendulum

$$T = 2\pi \sqrt{\frac{l}{g}} \tag{67}$$

we see that

$$C = \frac{2\pi}{\sqrt{g}} \tag{68}$$

or

$$g = \frac{4\pi^2}{C^2} . \tag{69}$$

From the mean value of C found in Exercise 23, compute the value of g. The value of g found at a place, H meters above sea level, may be *reduced to sea level* by means of the equation

$$g_0 = g + 0.0003086\, H \tag{70}$$

Compare this reduced value g_0 with the *normal* value γ_0 at sea level and latitude ϕ, given by Helmert's formula

$$\gamma_0 = 978.03\ (1 + 0.005302 \sin\phi - 0.000007 \sin^2\phi\ ) \tag{71}$$

FORM OF RECORD.

Exercise 24. To compute value of g.

Date.....

$C =$
$\log C =$ $\log 4 =$ .....
$\log \pi^2 =$ .....
$\operatorname{colog} C^2 =$ .....
$\log g =$ ..... $g =$ .....
$g_0 =$ .....

63. **Exercise 25. Moment of Inertia of a Connecting Rod.** The moment of inertia of a body about its center of gravity may easily be determined, provided the body can be vibrated as a physical pendulum from a point whose distance from the center of gravity is accurately known.

A connecting rod (Fig. 31) is well adapted for this experiment. The mass M of the rod is measured once for all and given to the student by the instructor. The center of gravity is located upon a line C which is found by balancing the rod upon a knife edge. This line should be carefully marked on the bar.

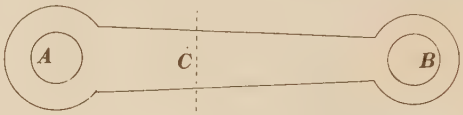

Fig. 31.

If the rod is now hung from a knife edge, placed inside the circular hole at either end, it forms a physical pendulum whose time of vibration is given by the equation

$$T = 2\pi \sqrt{\frac{I}{M h g}} \qquad (72)$$

where I is the moment of inertia about the point of suspension and h the distance from the point of suspension to the center of gravity of the rod.[1]

Let the rod be swung first about the point A whose distance h_1 from the line C must be accurately measured. Determine the period T_1 by the method of article 44. Then

$$T_1 = 2\pi \sqrt{\frac{I_1}{M h_1 g}}$$

where I_1 is the moment of inertia about the point A. From this equation I_1 is calculated.

Next, suspend the rod from point B whose distance from C is h_2. If the time of vibration in this case be T_2 we have

$$T_2 = 2\pi \sqrt{\frac{I_2}{M h_2 g}}$$

where I_2 is the moment of inertia about point B and is calculated from the last equation.

[1] *College Physics*, Article 55.

The moment of inertia about the center of gravity is[2]

$$I_0 = I_1 - M h_1^2 \qquad (73)$$

Similarly

$$I_0 = I_2 - M h_2^2$$

Calculate I_0 from both equations and take the mean as the final result.

FORM OF RECORD.

Exercise 25. To find the moment of inertia of a connecting rod about its center of gravity.

$M = \ldots\ldots$ $\qquad$ $h_1 = \ldots\ldots$
$g = \ldots\ldots$ $\qquad$ $h_1 = \ldots\ldots$
Determination of T_1 $\qquad$ Determination of T_2
(Record as in article 44)
$I_1 = \ldots\ldots$ $\qquad$ $I_2 = \ldots\ldots$
$I_0 = \ldots\ldots$ $\qquad$ $I_0 = \ldots\ldots$
Mean $I_0 = \ldots\ldots$

64. **Exercise 26. Moment of Inertia from Torsional Vibrations.** It is frequently of importance to determine the moment of inertia of a body whose form is irregular or such as to render its determination by computation difficult or impossible. In such case the moment of inertia may be determined by the method of **torsional vibrations.**

Suspend the body by a stout wire so as to swing freely about a vertical axis, with its surface horizontal. If twisted through an angle it will tend to return to its position of equilibrium, and in doing so will execute simple harmonic vibrations of period T. If I be the moment of inertia of the body and $\mathcal{T}_1$ the moment of the **restoring couple per unit angular twist,** then from equation (60) we have

$$T = 2\pi\sqrt{\frac{I}{\mathcal{T}_1}}.$$

If now there be added to the body a ring whose moment of inertia

[2] *College Physics,* Article 51; Eq. 134.

I_r, may be readily calculated from its dimensions, (Table VIII), then the period of the system becomes

$$T_2 = 2\pi \sqrt{\frac{I + I_r}{\mathscr{T}_1}} \tag{74}$$

Eliminating $\mathscr{T}_1$ from these two equations we have

$$I = I_r \cdot \frac{T^2}{T_1^2 - T^2} \tag{75}$$

Again if to the original system there be added a pair of cylinders, each of mass M, and radius r, symmetrically placed, each at distance a, from the axis of rotation, the period of the combined system becomes

$$T_3 = 2\pi \sqrt{\frac{I + 2I_c}{\mathscr{T}_1}} \tag{76}$$

where I_c, the moment of inertia of a single cylinder about the axis of rotation, is given by the equation[1]

$$I_c = \frac{M r^2}{2} + M a^2 \tag{77}$$

Combining equations (60) and (76) we find

$$I = 2I_c \cdot \frac{T^2}{T_3^2 - T^2} \tag{78}$$

Finally, if to the original system we add both ring and cylinders and determine the period of vibration T_3, of the system, we secure a third value for I in the same way as above, from the equation

$$I = (I_r + 2I_c) \cdot \frac{T^2}{T_3^2 - T^2} \tag{79}$$

The body to be investigated may have the form of a rectangular

[1] *College Physics*, Article 51.

bar as the magnet of a magnetometer, or the form of a circular cylinder or wheel, or it may be of any irregular outline, so long as it be furnished with a stout hook or other device for suspending it from an axis that shall pass vertically through its center of gravity. Its surface when suspended should be horizontal, and should have marked upon it lines to insure the exact placing of the ring and cylinders. The position of the centers of the cylinders should be clearly indicated in order to facilitate the determination of a. A line connecting these centers must pass through the axis of rotation of the body.

The ring should be of rectangular cross section, with sides and edges well polished, and a diameter clearly marked upon each. The cylinders should be accurately turned, as nearly alike as possible, and have sufficient mass to insure a distinct difference in the period of vibration when they are added. The ring should have a mass of at least one kilogram.

The exercise comprises the following measurements:

(a) Measure the external and internal diameters of the large ring by means of the vernier caliper and compute the external and internal radii, r_1 and r_2.

(b) Measure on the dividing engine the distance between the centers of the two cylinders and compute the length a, from the axis of rotation of the system to the center of either cylinder when placed upon the body.

(c) Measure the diameter of the cylinders and compute the mean radius r.

(d) Determine the mass of large ring M_r, and of either of the two cylinders M_c.

(e) Determine to thousandths of a second the time of vibration of the body alone, T; of the body and ring, T_1; of the body and cylinders, T_2; of the body, ring and cylinders, T_3.

FORM OF RECORD.

Exercise 26. To determine the moment of inertia of a body by torsional vibrations.

(*a*) Enter results as in Art. 44.

(*b*) Value		Value
M_r		T
r_1		T_1
r_2		T_2
I_r		T_3
M_c		$T_1 + T$
r_c		$T_1 - T$
a		.....
I_c		.....
		.....

Computation:

$$I = I_r \frac{T^2}{T_1^2 - T^2}$$

$\log M_r =$
$\log (r_1^2 + r_2^2) =$
$\operatorname{colog} 2 =$
$\log I_r =$
$\log T^2 =$
$\operatorname{colog} (T_1 + T) =$
$\operatorname{colog} (T_1 - T) =$
$\log I =$
$I =$

65. Exercise 27. The Ballistic Pendulum. When two bodies collide in such a way that their velocities at the instant of collision lie along the common normal at the point of contact we know from Newton's Third Law of Motion that the sum of the momenta of the two bodies is unchanged by the collision. If m_1 and m_2 be the masses of the two bodies, v_1 and v_2, v'_1 and v'_2 their respective velocities before and after collision, then

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \qquad (80)$$

Newton also showed that under the foregoing conditions the relative velocities, before and after impact, bore a constant ratio to each other, or if we assume v_1 to be the velocity before impact of the more rapidly moving body, we have

$$v'_1 - v'_2 = e\,(v_1 - v_2) \qquad (81)$$

where e is a quantity depending upon the material of which the bodies are composed. Investigations by Hodgkinson,[1] and by Vincent[2] show that e is influenced in some degree by the initial velocity. The quantity e is termed the coefficient of restitution, and is always less than unity.

In the laboratory the foregoing principles are easily verified by the study of the collisions between suspended spheres, where the direction of motion may be controlled and the respective velocities

[1] Hodgkinson, *Report of British Association*, 1834.
[2] Vincent, *Proc. Cambridge Phil. Society*, Vol. X, p. 332.

may be readily measured. For convenience of adjustment the spheres are taken of the same size, and the masses varied by making them of different material.

The apparatus (Fig. 32), consists of two spheres of wood, ivory, iron or lead, about 9 cms in diameter, hung on bifilar suspensions, about 250 cms in length, and so adjusted as to swing accurately parallel to the graduated arc ss'. The spheres carry at the lower ends of their vertical diameters small pointers by means of which their positions may be read off on the graduated arc. When at rest the spheres must touch lightly, and their centers lie on an arc parallel to the scale.

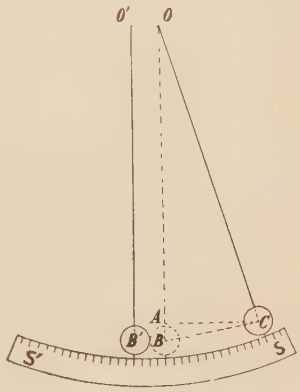

Fig. 32.

Read the position of the pointers, and displace one of the spheres, say the lighter one, through an arc of not more than 20°. Read off the displacement $BC = c$ accurately from the graduated scale, and allow the sphere to swing back against the one at rest. Determine as accurately as possible the extreme positions of the two spheres after impact, taking the mean of at least five simultaneous readings for each. Two observers are required.

Repeat the experiment by allowing the heavier sphere, raised through a smaller arc to swing against the lighter one and taking readings as before. In all cases take care to avoid giving a spinning motion to the released sphere. The masses of the spheres are to be determined by the student unless furnished by the instructor in charge.

From Fig. 32 it is seen that[1]

$$v_1^2 = 2g\,AB \tag{82}$$

and also

$$AB = \frac{c^2}{2\,OB} \tag{83}$$

whence

$$v_1 = c\sqrt{\frac{g}{OB}}, \tag{84}$$

[1] *College Physics*, Article 25.

and the velocity v_1, of the impinging sphere is directly proportional to the chord $CB = c$, of the arc through which it swings. For angles not exceeding 20° the arc may be set equal to the chord to within one-third of one per cent. We may therefore insert readings from the graduated arc in equation (80) instead of the actual velocities.

Apply equation (80), in which v_2 is to be taken equal to zero, and v_1, the velocity of the impinging sphere, is to be regarded as positive in each case, and the reverse direction as negative.

FORM OF RECORD.

Exercise 27. The ballistic pendulum. To prove the law of conservation of momentum.

Sphere Date.....

1 Material..... $m_1 =$ .....

2 Material..... $m_2 =$ .....

No.	v_1	v_2	v'_1	v'_2	$m_1v_1+m_2v_2$	$m_1v'_1+m_2v'_2$	Diff.	v_1-v_2	$v'_2-v'_1$	e

CHAPTER IV.

DENSITY.

66. **Definition.** Density is defined as **mass per unit volume,** and is therefore, in the C. G. S. system, measured by the number of grams per cubic centimeter. The International Bureau of Weights and Measures however, for practical reasons, chooses **the milliliter** as the unit of volume, where the liter is defined as the volume occupied by one kilogram of distilled water at 4°C (= 1000.05 cm^3). The density of water at this temperature becomes then strictly equal to unity, or one gram per milliliter.

According to this definition of unit volume, the numerical value for the density of any substance becomes identical with that denoting its specific gravity. In what follows no distinction will be made between the two definitions, since the degree of accuracy attainable by the methods described below, does not warrant such a distinction.

67. **Exercise 28. Density from Mass and Volume.**

FORM OF RECORD.

Exercise 28. To determine the density of a brass cylinder from its volume and mass.

Density of brass cylinder No..... Date.....

Length	Diameter	
.....	.....	Mass.....
.....	.....	
.....	.....	
.....	.....	
.....	.....	
Mean.....	Mean.....	
	Volume.....	Density.....

68. **Exercise 29. The Pyknometer.** The **pyknometer** in its simplest form consists of a glass vessel (Fig. 33), provided with

Fig. 33.

an accurately ground stopper, perforated throughout its length. Before use it is to be thoroughly cleaned and dried with alcohol or ether. It is then carefully weighed. Call this weight W_0. The pyknometer is then filled with distilled water, placed in a bath of water at 30°C. and allowed to remain there five minutes. (Why?) It is then wiped dry, and its weight, W_1, determined. After cleaning and drying as before, the pyknometer is filled with the liquid under examination, brought to the temperature of 30°C, wiped dry and weighed. Call this weight W_2. Derive and apply formula for the density of a liquid. Correct for temperature, by multiplying d_0, the value found, by d_1, the density of the standard[1] (for water at 30°C, $d_1 = 0.9958$ g per cc). Unless the values are to be carried to more than three decimal places, the buoyant force of the air may be neglected. Instead of making all the weighings at 30°, any other temperature may be taken provided it be higher than the temperature of the room where weighings are made (Why?)

FORM OF RECORD.

Exercise 29. Determine the density of two liquids.

Pyknometer No..... Date.....

Density of.....

$W_0 =$ .....

$W_1 =$ ..... $d_0 =$ .....

$W_2 =$ ..... $d_1 =$ .....

Corrected density, d, of..... = ...

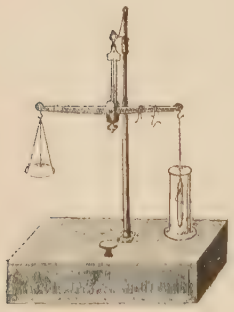
Fig. 34.

69. **Exercise 30. Mohr's Balance.** In **Mohr's balance** (Fig. 34), the beam is divided into ten equal parts of which the last coincides with the end. Upon this end is hung by means of a fine platinum wire a small sinker containing a **thermometer.** The weights consist of four pairs of riders weighing respectively m, 0.1 m, 0.01 m, 0.001 m grams. The instrument is usually so adjusted that the sinker is exactly counterpoised in air. When immersed in water

[1] *College Physics,* Article 69.
Also Stewart and Gee, *Practical Physics,* Vol. I, p. 119.

at 4° C the buoyant force on the sinker is equal to the weight of the largest rider. When the sinker is put into any other liquid the weights needed to equal the buoyant force upon the sinker in each case are in direct proportion to the densities of these liquids. If m be called unity then the density of the liquid can be read directly from the beam. The balance must show for water at $t°$ C the density corresponding to this temperature as given in Table II. If, instead of this density d, it show a density d_1, then all results must be multiplied by d/d_1. It is obvious that the instrument may be used as an ordinary balance as well, and the densities of solids and liquids referred to water at any temperature may be determined by means of it with equal ease.

FORM OF RECORD.

Exercise 30. Mohr's balance. Redetermine the densities of the substances used in Exercises 28 and 29.

1. For solids: Date.....
   Density of.....
   Weight in air.....
   Weight in water..... Temperature of water.....
   d_1.....
   Corrected density.....

2. For liquids: Date.....
   Density of.....
   Temperature .....
   Buoyancy of sinker in water.....
   Buoyancy of sinker in liquid.....
   Density.....

CHAPTER V.

SURFACE TENSION AND VISCOSITY.

70. **Characteristics of a Liquid.** A **liquid** is a body which has **no elasticity of shape,** or which **yields continuously under** the action of a **shearing stress.** It is characterized by considerable **mobility of** its **molecules,** by a **distinct, free upper surface,** usually of the meniscus shape when confined in a tube, and by the existence in that **free surface** of a **specific stress or tension** not found elsewhere in the body. As a result of this tension the liquid behaves as if it were enclosed in an elastic bag which tends to contract indefinitely and compress the liquid into as small a volume as possible.

If the liquid exist in the form of a film then the two sides of the film exhibit this tension in like degree and the film tends to contract indefinitely unless prevented by the application of external force. If the external force required to keep a film, l units in width, in equilibrium be F dynes, then T, the **surface tension** of the film, measured in dynes per centimeter, is given by the equation[1]

$$2\,T\,l = F \tag{85}$$

or

$$T = \frac{F}{2\,l} \tag{86}$$

As the surface tension of any liquid is greatly modified by the presence of any substance dissolved in it, it is of the highest importance in experiments on surface tension that all surfaces coming in contact with the liquid be absolutely clean, and that the liquid itself be pure.

71. **Exercise 31. Measurement of Surface Tension.** The apparatus consists of a Jolly balance and several small frames of

[1] *College Physics,* Article 89.

different width made from short pieces of fine glass tubing or fine platinum wire, with the ends bent so as to form three sides of a rectangle, all lying in the same plane. Suspend one of these frames from the spring of a Jolly balance by a fine thread or wire after removing the pan, and adjust so that the sides of the frame hang vertically and the ends just dip into the liquid. Take the reading on the Jolly balance, making at least three adjustments and taking the mean. Next form the film by raising the vessel containing the liquid until the horizontal side of the frame is immersed and lowering the vessel again to its former position. The tension in the film tends to pull the frame back into the liquid and elongates the spring. Raise the frame by the rack and pinion movement until the indicator has reached its former zero position. Note the reading. Make four determinations with different lengths of the film for each frame, using frames of three different widths.

Determine the constant of the spring of the balance by placing gram masses upon the pan and noting the elongation per gram. Calculate the force $m\,g$, in dynes exerted by the spring of the balance to hold the film in equilibrium. The surface tension of the film in dynes per centimeter width, is given by

$$T = \frac{m\,g}{2\,l} \qquad (87)$$

FORM OF RECORD.

Exercise 31. To determine the surface tension of water, and of a soap solution at room temperature.

Temperature..... Date.....

Constant of Jolly Balance		Width of Frame	Zero	Readings	
Mass	Reading			Water	Soap Solution
		1			
		2			
		3			
		4			

Force in dynes to stretch spring one scale division, = ...

Surface Tension		
Frame No.	Water	Soap Solution
1		
2		
3		
4		
	Mean	

72. **Exercise 32. Surface Tension from Capillary Action.** If a freshly drawn glass tube of small diameter be held vertically and lowered into clean water, the surface action of the water drags the liquid up the tube to a height h, at which the weight of the liquid column just balances the vertical component of the force due to surface tension, and equilibrium ensues. If r be the radius of the tube and θ the angle of contact between the liquid and the tube and d the density of the liquid, then for equilibrium we have

$$2\pi r T \cos\theta = \pi r^2 h d g + v d g \tag{88}$$

where v is the volume of the liquid forming the meniscus.

If the liquid wet the tube θ may be set equal to zero, and if we assume the radius to be so small that the cup formed by the meniscus is hemispherical, then

$$v = \frac{\pi r^3}{3} \tag{89}$$

and the expression for T becomes

$$T = \frac{r d g}{2}\left(h + \frac{r}{3}\right) \tag{90}$$

or

$$T = \frac{r d g h'}{2} \tag{91}$$

where h' *is* the distance from the surface of the liquid to a point $r/3$ above the lowest point of the meniscus.

Place a freshly drawn glass tube, of less than one millimeter diameter, in a glass cup with straight sides and pour in two or three centimeters depth of distilled water. Move the tube up and down through a range of a centimeter or two so as to make sure that the meniscus is formed against a wet surface. Measure with a finely divided scale or better with a micrometer cathetometer, the height h'. Mark the point on the tube reached by the water column and with a fine file cut the tube at this point. Measure the diameter of the tube with the micrometer microscope, taking the mean of at least three readings. Note the temperature and insert in the formula the appropriate value for d. Compute the value of the surface tension for pure water at the observed tem-

perature. Repeat the experiment with a dilute solution of fuchsine in alcohol.

FORM OF RECORD.

Exercise 32. To determine the surface tension of distilled water and of an alcoholic solution of fuchsine at room temperature.

Temperature.....	Date.....
Readings	Diameter of tube
Surface of liquid Meniscus	1
$d =$	2
T for water = ...	T for fuchsine solution = ...

73. **Coefficient of Viscosity.** As has been shown in a previous article, a liquid yields continuously under the action of a shearing force. An ideal liquid would offer no resistance to such a force but would suffer its molecules to glide past each other without any loss of energy. Most liquids however offer some resistance to a shearing force and this property by virtue of which a liquid resists the relative motion of its parts is termed viscosity.

The **coefficient of viscosity** is defined as the constant ratio of the shearing stress in a fluid to its time rate of change of shearing strain. The latter is given by l/Lt, equal to v/L. If F be the force acting on one of two parallel surfaces A of a fluid, separated by a distance L cm and moving with a velocity v relative to each other, the coefficient of viscosity of the fluid is

$$\gamma = \frac{\frac{F}{A}}{\frac{v}{L}} = \frac{F\,L}{A\,v} \qquad (92)$$

The coefficient of viscosity is therefore numerically equal to "the tangential force on unit area of either of two horizontal planes at unit distance apart, one of which is fixed, while the other moves with the unit of velocity, the space between being filled with the viscous substance."[1]

This quantity may be measured either by determining the flow of a viscous fluid through a tube of small bore, or by determining the resistance offered by the fluid to the passage of a solid body through it.

[1] Maxwell, *Theory of Heat.*

74. **Exercise 33. Coefficient of Viscosity by Flow through a Capillary Tube.** It is shown in works on physics[1] that the volume of a liquid discharged in time t, through a capillary tube of radius r, and length l, under a pressure of p dynes per square centimeter, is given by the equation

$$V = \frac{\pi p r^4}{8 \eta l} t = \frac{\pi h d g r^4}{8 \eta l} t, \qquad (93)$$

where V is the volume, d the density, η the coefficient of viscosity of the liquid, and h the height from which it flows. From this it follows that

$$\eta = \frac{\pi p r^4}{8 V l} t = \frac{\pi h d g r^4}{8 V l} t = \frac{\pi h d^2 g r^4}{8 m l} t, \qquad (94)$$

when m is the mass of the liquid discharged through the tube.

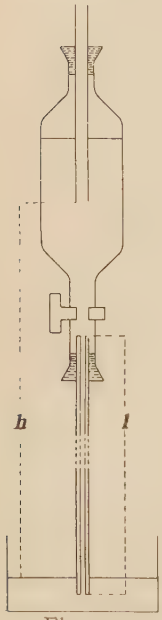

Fig. 35.

To determine this coefficient in absolute measure, each of the above quantities must be expressed in absolute units.

Select a capillary tube of uniform bore and clean it thoroughly by running through it repeatedly a solution of sodium bi-chromate and strong sulphuric acid, and afterward washing it well with distilled water. Fasten the tube, by means of a rubber stopper, to the lower end of a large separating flask, as shown in Fig. 35.

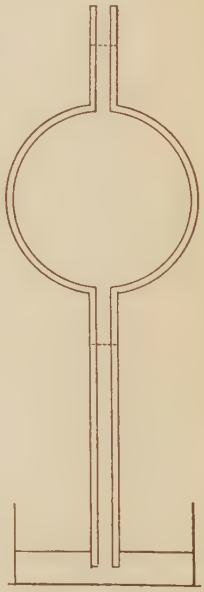
Fig. 36.

Pour in some of the liquid to be studied and allow it to flow through the capillary for a short time. Now close the stopcock,

[1] Poynting and Thompson, *Properties of Matter*, Chapter XVIII.

fill the flask with the liquid under examination, and arrange for constant head during the flow, by inserting in the upper end of the flask a stopper through which passes a piece of glass tube of wider bore, reaching well down into the liquid as shown in the figure.

Place under the lower end of the capillary a wide beaker, or flat dish, and pour in sufficient liquid to cover the end of the capillary tube. Weigh this vessel with its contents. Replace the beaker, open the stopcock and allow the liquid to flow for a definite time t. Weigh the beaker and its contents a second time and determine m, the mass of liquid discharged. Determine the density of the liquid if necessary by means of Mohr's balance, the lengths l and h, by means of a meter stick and the radius of the tube by filling a measured length of it with mercury, weighing it, and computing the average radius from the length, mass, and density of the mercury. Use three capillary tubes of different radii.

For a determination of the specific viscosity, *i. e.*, the ratio between the viscosity of the liquid and that of water at 0°C, or at room temperature, the dimensions of the capillary tube need not be known. By performing the experiment, with the apparatus unchanged, first with water and afterward with the liquid, we have

$$\frac{\eta_1}{\eta_2} = \frac{d_1^2 m_2 t_1}{d_2^2 m_1 t_2}, \tag{95}$$

or

$$\frac{\eta_1}{\eta_2} = \frac{d^2_1 m_1}{d^2_2 m_1} \tag{96}$$

if t, the time of flow be made the same in each case.

The exercise may be varied by allowing equal volumes of the two liquids to flow through the capillary, and comparing the times required, from which we have

$$\frac{\eta_1}{\eta_2} = \frac{d_1 t_1}{d_2 t_2}. \tag{97}$$

In this case a tube as shown in Fig. 36 is very convenient. Draw the liquid into the tube until it rises above the upper mark.

Then let it run out and note the time it takes from the instant it passes this mark until the liquid enters the capillary. The time may be determined conveniently by means of a stop watch.

FORM OF RECORD.

Exercise 33. To determine the coefficient of viscosity of water at room temperature.

Temperature of water..... Date.....

No. of tube	1	2	3	No. of tube	1	2	3
Length				Time of flow			
Mass of mercury				Mass of beaker after			
Volume of tube				Mass of beaker before			
Average radius				Mass of water			

$\eta_1 =$

$\eta_2 =$

$\eta_3 =$

Mean $=$

CHAPTER VI.

MEASUREMENTS IN SOUND.

75. **Exercise 34. Velocity of Sound in Metals. (Kundt's Method.)** A brass rod held firmly clamped at its middle point when stroked with a resined cloth vibrates longitudinally like the air in an open organ pipe when sounding its fundamental tone. The middle of the rod being rigidly fixed is obviously a node and the length of the rod is therefore **the half wave-length in brass** of the sound produced. If the end of the rod be brought into contact with an enclosed column of air whose length may be varied at will, it is possible so to adjust the length of the air column as to render it capable of vibrating in unison with the rod. In this case the enclosed air column having been thrown into stationary vibration behaves as a resonator closed at both ends; it must therefore contain at least one, and usually contains a number of **half wave-lengths,** of the **sound in air,** produced by the rod. From the fundamental equation connecting velocity, frequency and wave-length, we have[1]

$$N = \frac{V}{\lambda} = \frac{v}{\lambda'}. \tag{98}$$

where N is the frequency and V, v, λ, and λ' denotes the velocities and wave-lengths of the same sound in brass and in air respectively. From this we have at once

$$V = v \cdot \frac{\lambda}{\lambda'} \tag{99}$$

where v, the velocity of sound in air must be corrected for temperature according to the formula[2]

$$v_t = v_0 \sqrt{(1 + at)} \tag{100}$$

where a for air at ordinary humidity, may be taken as 0.004 per degree C.

---

[1] *College Physics,* Article 104.
[2] *College Physics,* Article 114.

In practice a brass rod about one centimeter in diameter and one meter long is clamped in a vise at its middle point and bears at one end a small disk of paper. A glass tube about 5 cms in diameter and 150 cms long (Fig. 37), has one end closed air-tight by a sheet of rubber membrane tied smoothly over the end, while the other end is furnished with an adjustable piston sliding freely in the tube. The walls of the tube are lightly dusted throughout with fine cork filings or amorphous silica. The tube is placed horizontally upon two V-shaped wooden supports so that the rubber membrane presses lightly against the disk of stiff paper on the end of the rod. The rod is set in vibration by chafing it gently with a piece of cloth or chamois skin covered with powdered resin. The cloth or chamois skin should be held between the thumb and fore finger of each hand and pressed lightly against

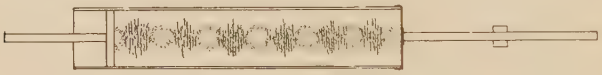

Fig. 37.

each side of the rod. When the rod is properly clamped a very slight pressure is sufficient to produce a loud, clear tone. Adjust the piston in the outer end of the tube until the enclosed air vibrates freely on stroking the rod. This is indicated by the powder being tossed about in the tube and falling in the characteristic figures shown above. The nodes are indicated by small rings of powder and the antinodes by transverse layers or striae. The value of $\lambda'/2$ is found by measuring over a number of circles from center to center, and dividing the distance by the number of spaces or loops measured, remembering that the distance from node to node is equal to $\lambda'/2$. It will be noticed that a node is found both at the rubber diaphragm and at the piston. Why?

Measure over as large a number of loops as possible and compute $\lambda'/2$. Tap the tube lightly, rolling it over and over until the powder is evenly distributed, and repeat the determination. Take at least five separate sets of measurements. Avoid heating the rod by undue pressure or by continued rubbing.

FORM OF RECORD.

Exercise 34. To determine the velocity of sound in brass.

Temperature..... Length of rod..... Date.....

Number of loops between nodes	Distance	$\frac{\lambda'}{2}$	$\frac{\lambda}{2}$ = ...
.....	.....	.....	$\frac{\lambda}{\lambda'}$ = ...
.....	.....	.....	
.....	.....	.....	
.....	.....	.....	v = ...

Mean.....

Velocity of sound in brass = ...

76. **Exercise 35. Computation of Young's Modulus.** From the equation for the velocity of sound in any medium

$$V = \sqrt{\frac{e}{d}} \qquad (101)$$

where c is the **coefficient of elasticity,** and d the **density** of the medium in question, we may compute c at once from the data obtained in previous experiments. In the case of longitudinal waves transmitted through solids the coefficient of elasticity involved is M, **Young's modulus,** whence

$$M = V^2 d . \qquad (102)$$

FORM OF RECORD.

Exercise 35. To compute Young's modulus for brass from velocity of sound and density.

Date.....
V as found in Exercise 34 = ...
d as found in Exercise 28 = ...
M = ...

Compare result with that obtained in Exercises 18 and 20.

77. **Exercise 36. Rating a Tuning Fork. Graphical Method.** A **tuning fork,** one prong of which is armed with a fine sharp tip of flexible sheet copper, is mounted at right angles to a

metallic cylinder. The cylinder is carried upon an axis furnished with a thread so that when rotated it is advanced longitudinally at the same time, so that the tracing point generates a spiral upon the surface of the cylinder. If the surface of the cylinder be slightly smoked, the tuning fork set in vibration and the cylinder rotated uniformly the tracing point describes a sinusoidal curve upon its surface. The **number of vibrations executed** in a second being thus **automatically recorded** by the fork, it is only necessary to indicate the beginning of the successive seconds upon the curve in order to read the vibration frequency of the fork directly from the surface of the cylinder.

In practice the cylinder is covered with a sheet of firm smooth paper pasted smoothly on by gumming one end of the paper and pressing the gummed surface upon the other. *The paper must not be stuck upon the cylinder.* The paper is then smoked uni-

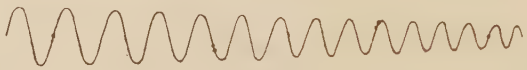

Fig. 38.

formly and lightly by means of a gas flame passed back and forth near the paper while the cylinder is continuously rotated, allowing only about an inch of the tip of the flame to touch the paper.

Care should be taken not to smoke the paper too black. The fork is then adjusted in its holder so that the point just touches the paper at the highest part of the cylinder and at the left end of the cylinder so that when the latter is rotated the fork seems to move from left to right along the cylinder. *The cylinder must rotate from the tracing point.* The time intervals are recorded upon the paper by connecting the cylinder and the fork to the secondary terminals of an induction coil, the primary circuit of which contains a suitable battery and is closed by a pendulum beating seconds. Consequently when the tracing point rests upon the paper and the coil is put in action a spark passes from the point to the cylinder each time the circuit is closed, *i. e.,* every second. The passage of this spark leaves a small spot on the smoked surface thus marking the time very accurately. The appearance of the sparks should be like that shown in Fig. 38.

Sometimes the coil will give two or three sparks instead of one. (Why?) In this case read *from the last one.* The fork should be so adjusted that when the point touches the paper and the fork is properly bowed it will continue to vibrate for at least twelve seconds before coming to rest. When all the adjustments are made the coil is put in action, the fork bowed and the cylinder rotated uniformly while the fork continues to vibrate. The fork is then bowed again and the cylinder rotated as before. When the paper is filled the fork is removed, *the paper cut along the lap,* parallel to the axis of the cylinder almost but not quite apart; the cylinder is then turned over and the paper broken loose. In this way only can the paper be removed from the cylinder without spoiling the record.

The paper is then passed, face upwards, through a fixing solution of shellac in alcohol and then dried. In a few minutes the curve is ready to be examined. Count the waves for ten seconds and record. In counting the waves always take an **even number** of seconds.

FORM OF RECORD.

Exercise 36. To determine the frequency of a tuning fork by the graphical method.

Date.....

Seconds	:	Number of Vibrations	:	N

CHAPTER VII.

MEASUREMENTS IN HEAT.

78. **Effects of Heat.** In general, all physical properties of a body, except its mass and weight, are changed by adding to it **heat energy.** In this chapter only the more distinctive of such changes will be considered, among which the following may be noted:

(*a*) The **temperature** of the body **rises.**

(*b*) The body undergoes a **change in volume**; in general an **increase in volume** of the body **attends** an **increase in temperature,** if the **pressure** remain **constant.**

(*c*) The body exhibits a **change of pressure,** if its **volume** be kept **constant.**

(*d*) The body may **change its state or condition,** as for example, ice changes to water and water to steam upon the **addition** of **definite quantities of heat.**

Other changes, such as changes in the **molecular, optical, electrical** or **magnetic properties** of a body, incident upon the **addition of heat energy** to it, are usually investigated only in so far as such changes are dependent upon **changes** of **temperature.** The **percentage variation** of any physical property of a body **per unit change** of **temperature** is termed **temperature coefficient,** and is more or less **characteristic of the substance** of which the body is composed. The determination of such characteristic coefficients falls more naturally under the chapters dealing with those properties.

THERMOMETRY.

79. **Thermometry.** If two bodies possessing different temperatures be brought into thermal union, heat flows from the one

of higher temperature to the one of lower temperature, and in general the flow of heat is such as to produce and maintain an equilibrium of temperature in the body, or system of bodies, if thermally insulated from all other bodies.

Temperature is defined quantitatively in terms of the increase in pressure of hydrogen gas of constant volume, under the assumption that equal increments of temperature produce equal increments of pressure in the gas. As points of reference in the measurement of temperature the two fixed points for water, the freezing and the boiling points under standard conditions, have been chosen. In the centigrade scale the temperature of melting ice is called 0°, and the temperature of steam forming freely under a pressure of 760 mm of mercury, is taken as 100°.

Temperatures are usually measured by means of mercury-in-glass thermometers. Such thermometers possess numerous disadvantages as compared with the gas or air thermometer, prominent among which are the following:

(*a*) Inequality of the bore of the glass tube.

(*b*) Inequality of the scale.

(*c*) Neither glass nor mercury expands equally and uniformly throughout any large range of temperature.

(*d*) The instability and uncertainty of the fixed points of the thermometer, arising either from slow changes going on in the thermometer itself or from sudden and large variations in temperature incident upon the use of the thermometer.

From these causes it is evidently a matter of first importance to verify the readings of a mercury-in-glass thermometer, with which any accurate work is to be attempted.

80. **Exercise 37. Determination of the Fixed Points of a Thermometer.** The fixed points of an ordinary thermometer are usually in error by some fraction of a degree and these errors when determined, form the basis of correction to be applied to all subsequent readings made with the instrument. The fixed points

must be frequently determined. These determinations fall under two heads:

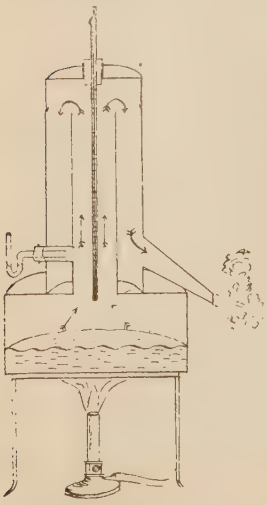

Fig. 39.

(*a*) *The Boiling Point.* The apparatus (Fig. 39), consists of a brass vessel partly filled with water and having in its upper part double walls so arranged that the steam passes up through the inner cylinder, down through the outer space and escapes from a short tube near the bottom. The thermometer is passed snugly through a close-fitting cork, into the inner cylinder. The bulb is not allowed to come in contact with the water and should be surrounded by **wire gauze** to prevent overheating. If possible, almost the entire filament of mercury should be enclosed by the steam issuing freely under atmospheric pressure. The temperature of the steam is found from the barometric pressure under which the water boils, by means of the following approximate formula

$$t = 100° + 0.0375\,(B - 760), \qquad (103)$$

where B denotes the barometric pressure in mm, corrected to 0° C. For accurate values of boiling point at different pressures see Table IX.

If the thermometer reads $t + b$, instead of the calculated t degrees, the correction to be applied to this reading is $-b$ degrees.

No readings should be taken until the steam issues freely from the tube at the bottom. This tube should be kept entirely open and the water should not be boiled too violently. (Why?)

(*b*) *The Zero Point.* The thermometer is now removed from the boiling point apparatus and as soon as the temperature has dropped to 50° is plunged into a clean vessel containing a mixture of distilled water and pure ice so that the mercury thread is entirely surrounded by the mixture. When the reading is taken

the thermometer should be raised just high enough to show distinctly the **upper end** of the **mercury filament.** In reading care should be taken to **avoid parallax.** As soon as the mercury has fallen to 1 , note the reading every minute until it has remained stationary for five minutes. Take the final readings as the zero reading of the thermometer.

What would be the correction for the zero point, if the reading is found to be $+ a$ degrees?

The zero point thus found is termed the **depressed zero,** since it is usually lower than the value found for the zero point, if it be determined just before the boiling point is taken. This difference is due to the fact that the **glass contracts slowly** for a considerable time after being heated and allowed to cool, and is thus unable to follow immediately sudden changes in temperature. The **depressed zero point** is, however, the one to be **used for calibration.**

Under the supposition that the bore of the tube and the scale are uniform between the two fixed points, the value of a scale part may be found in terms of the mercury in glass scale by dividing t degrees by $(t + b - a)$. Calculate by this method the value of the tenth, twentieth, thirtieth, etc., divisions of the scale. Compare the values so found with a calibration obtained by direct comparison of the thermometer with a standard thermometer.[1]

For this comparison both thermometers should be placed in a large bulk of water in such a way as to keep their bulbs close together, and the water constantly stirred during the comparison. Change the temperature of the water by steps of 5° at a time from 0° up to 50°, and read both thermometers each time after the water has been well stirred. Compare the corrected table obtained by the comparison with the standard, with the calibration obtained by calculation.

For **accurate determinations of temperature** it is necessary to **determine** the **depressed zero** point **immediately** after **taking** the **temperature** in question. The **temperature** must be **reck-**

[1] For the reduction of the mercury in glass scale to the normal (hydrogen) scale see Chappuis, Rapp. Congr. internat. Paris, 1900, Vol. I, p. 142.

oned from this depressed zero, even though it is not a constant for different temperatures.

Determine the fixed points of an ordinary thermometer and calibrate it from 0° to 50° C.

FORM OF RECORD.

Exercise 37. To determine the fixed points of a thermometer.

Thermometer No..... Date.....

Boiling point			Zero point		Calibration	
Reading	t	Correction	Reading	Correction	Compared	Computed
.....	....	.....	.....	.....	.....	.....
.....	....	.....	.....	.....	.....	.....

Value of one scale part =

81. Stem Correction. In order that the reading of the thermometer may give the exact temperature of the body in question, it is necessary that the entire mass of mercury should be at the temperature of the body. Frequently a portion of the filament extends above the region whose temperature is sought, and the reading will be too high or too low according as the temperature of the filament is above or below that of the body to be measured. The **relative coefficient** of **expansion** for **mercury in Jena** normal **glass** is 0.000157 per degree C. If $t°$ of the filament are outside, then the difference in height of the filament for a difference of 1° C will be 0.000157 $t°$. If $t°_0$ be the mean temperature of the filament exposed, and t_1 the temperature indicated by the thermometer, then the correction to be added to the indicated reading is 0.000157 $(t_1 - t_0)$ t degrees; t_0 is determined by an auxiliary thermometer.

EXPANSION.

82. Coefficient of Linear Expansion. The coefficient of linear expansion of a substance is defined as the increase per unit length, per degree increase of temperature. Thus if the substance be in the form of a rod or wire, and l_1 be its length at temperature t_1, and l_2 its length corresponding to t_2, then β, the coefficient of linear expansion, is by definition

$$\beta = \frac{l_2 - l_1}{l_1 (t_2 - t_1)} \qquad (104)$$

If one of the temperatures be taken as 0° C, the corresponding

length l_0, and the length at any other temperature t, be l_t, then the above formula reduces to

$$\beta = \frac{l_t - l_0}{l_0 t} \tag{105}$$

or

$$l_t = l_0 (1 + \beta t) \tag{106}$$

83. Exercise 38. **Coefficient of Linear Expansion of a Solid.** The solid whose coefficient of expansion is to be determined is in the form of a long thin rod R, Fig. 40, which is held

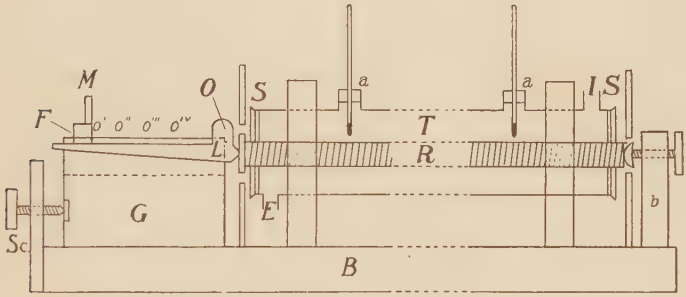

Fig. 40.

by means of thin rubber stoppers S S, at the center of a brass tube T, about five centimeters in diameter, and of approximately the same length as the rod. The tube is furnished with inlet and outlet tubes for the passage of water or steam. Thermometers are placed at the ends of the tube in two openings a a. The tube rests on two V-shaped blocks, which together with the block b, are fastened to the stout board B. The block b carries an adjustable screw whose axis is in line with that of the rod, and which has on its inner end a small glass cap, to prevent as far as possible any loss of heat from the rod by conduction. The screw is turned till the glass cap presses firmly against the end of the rod, and thus acts as the fixed point from which the length of the rod is measured.

The variations in length are measured by means of a series of

levers pressing against the other end of the rod. The lever L, mounted in a slot in the block G, rotates about O, and actuates the tilting mirror M, through the right angled arm F. Four shallow grooves, o', o'', o''', o^{IV}, are cut in the surface of the block at right angles to the level L, in which the points of the tilting mirror M, may be placed so as to stand astride of the slot. The rotation of the mirror is read off by means of a telescope and a vertical scale.

Loss of heat by conduction from the rod is prevented by inserting a small piece of glass between it and the point of the lever L, while the tube is covered with a layer of asbestos, and screens of asbestos are placed at either end to prevent loss by radiation. By means of the screw Sc, the block with its system of levers is pushed forward and the apparatus adjusted so that, with ice water flowing through the tube, the mirror stands vertically, and the zero reading is taken.

If the distance between O and the point of contact of the lever with the glass plate be called l_1, and l_2 be the distance between O and F, then when the rod expands by an amount a, for a difference in temperature t, the lever will rotate through an angle ϕ, and its ends will describe short arcs a and b, such that

$$a = \phi\, l_1 \tag{107}$$

and

$$b = \phi\, l_2 \tag{108}$$

Also, since the point F moves upward through the short arc b, the mirror is tilted through an angle θ, such that

$$b = \theta\, \rho \tag{109}$$

where ρ is the distance $o'F$.

By eliminating b and ϕ from the above equations we have

$$\theta = \frac{a\, l_2}{\rho\, l_1}. \tag{110}$$

Since the quantity $l_2/\rho\, l_1$ is a constant of the instrument we may write

$$\theta = k\, a \tag{111}$$

The angle θ is evaluated from the reading a, observed in the

telescope, and the distance D, between the telescope and the mirror by means of equation (18).

$$\tan 2\theta = \frac{a}{D}.$$

Since k, a, and D are known, the value of a may be readily determined, and from the relation

$$l_t = l_0 + a = l_0(1 + \beta t) \tag{112}$$

we have

$$\beta = \frac{a}{l_0 t}. \tag{113}$$

Determination of k. To determine the constant k, put glass slides of known thickness p, measured by means of the spherometer, between the end of the rod and the lever and observe the resulting deflection, while the temperature and the position of the block G, remain unchanged. Solve for k from the relation

$$k = \frac{\theta}{p}. \tag{114}$$

Use two different slides and two positions of the mirror. Since for accurate determinations of k, it is desirable that the glass slides should not be too thin, the telescope and scale should be brought nearer to the mirror, in order to keep the readings on the scale. In determining the deflection during the experiment, the sensitiveness may be increased at will by increasing the distance between the mirror and scale.

The temperature of the rod is changed by passing water of the desired temperature from a larger vessel through the tube. As soon as the temperature becomes constant, as indicated by the constancy of the scale reading, take this reading and that of the thermometers. Thermometer readings must be corrected for the exposed filament. Change the temperature of the rod by steps of about 20° from 0° to 100° C. In order to obtain the last point steam is passed through the tube and the barometer reading noted. Since the change in length is a very small fraction of the

original length of the rod, this length may be determined with sufficient accuracy by means of a millimeter scale.

Since in the instrument as described above a small portion of the rod at each end is not at the exact temperature of the water or steam, a slight error will be introduced, especially if the difference between the temperature of the rod and that of the room be large. The determination may be made much more accurate by holding the rods, which are cut slightly shorter than the tube, in the center of the tube by wire frames, and inserting in the rubber stoppers short pieces of nickel-iron whose coefficient of expansion is almost negligible.

In a similar type of apparatus the block with the system of levers is replaced by a spherometer screw, passing through a nut on a support solidly fastened to the base of the instrument. The end of the screw is brought into contact with the movable end of the rod and the position of the screw is read from the spherometer scale. Readings are repeated with the rod at different temperatures. From these readings the expansion of the rod can easily be calculated. The form of record, given below, must of course be altered correspondingly, if this type of apparatus be used.

FORM OF RECORD.

Exercise 38. To determine the coefficient of linear expansion of a metal and of glass.

Date.....

I. Determination of k.

Thickness of glass slide....	a	D	$\tan 2\theta$	θ	k

II. Determination of β. Length of rod.....

Temperature	a	D	$\tan 2\theta$	θ	α	β

84. **Expansion of Liquids.** In determining the coefficient of expansion of a liquid, we are met by the difficulty that the liquid must be contained in some sort of a **receptacle**, the material of which expands at the same time as the liquid, and hence the **apparent expansion** of the liquid is always a **differential effect,**

being the difference between the increase in volume of the receptacle and that of the expanding liquid. By measuring the height of two communicating columns of the same liquid at different temperatures the coefficient of cubical expansion of the liquid may be determined without reference to the expansion of the containing vessel. For the attainment of accurate results, however, the apparatus becomes too complicated for use in an elementary course[1].

It is therefore customary to measure the **relative expansion** of the liquid in a vessel, usually of glass, and calculate a the **absolute coefficient** of expansion of the liquid from this value and from the known coefficient of cubical expansion g of the vessel. This latter coefficient is found either by determining β, the linear coefficient for the glass of which the vessel is made, and putting $g = 3\beta$, or by using a liquid whose absolute coefficient is known. Thus let

a be the relative coefficient of cubical expansion of the liquid in glass,
a its absolute coefficient,
g the coefficient of cubical expansion for glass,
V the apparent volume of the liquid at $t°$ C,
V_1 the real volume at $t°$ C,
V_0 its volume at 0° C.

Then we have the following relations:

$$a = \frac{V - V_0}{V_0 t}, \tag{115}$$

$$a = \frac{V_1 - V_0}{V_0 t}, \tag{116}$$

and

$$g = \frac{V_1 - V}{V t}, \tag{117}$$

from which by eliminating V_1 and t, we have

$$a = a + g \frac{V}{V_0}, \tag{118}$$

or, since V/V_0 is very nearly unity

$$a = a + g. \tag{119}$$

[1] Preston, *Theory of Heat*, p. 170.

85. **Exercise 39. Coefficient of Expansion of a Liquid by the Dilatometer.** A dilatometer consists of a bulb with an accurately graduated stem of uniform capillary bore. In work of extreme precision it is necessary to calibrate the stem throughout in order to correct for irregularities of cross section. The volume of the bulb at 0° C, and that of one scale division on the stem must be accurately determined, as well as the coefficient of cubical expansion of the bulb. These form the constants of the instrument. The exercise is divided into three parts:

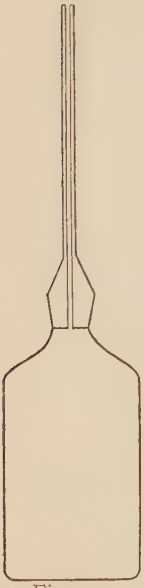

Fig. 41.

(*a*) *Determination of the volume of one division of the capillary stem.* Bring into the graduated stem an amount of mercury sufficient to fill it nearly full, and measure the length of the filament in terms of the scale divisions, using a mirror scale or reading microscopes. Let the observed length of the filament be l scale divisions. On account of the meniscus the length measured from end to end will be slightly too large. For capillary tubes it is sufficient to subtract from the observed reading $0.4\,h$, where h is the height of the meniscus in terms of a scale division.

Let the mass of the mercury filament be m grams, at the temperature $t°$. One gram of mercury weighed in air, occupies at $t°$ C, the volume

$$v' = 0.07355\ (1 + 0.000181\ t)\ cm^3 \qquad (120)$$

so that the volume corresponding to one scale division is

$$v = m\,\frac{v'}{l}.$$

(*b*) *Determination of the volume of the bulb.* The volume of the bulb may be determined by taking the mass of the dilatometer, first when empty and dry, and secondly when filled with air-free water up to scale part l' in the capillary tube, at a

temperature not below 15° C. Let the mass in the first case be m_0, and in the second m_t, then m, the mass of the water, is

$$m = m_t - m_0$$

One gram of water, weighed with brass weights in air, occupies very nearly $(2.00106 - d)$ cm³, where d is the density of the water at temperature $t°$, as given in Table II. The volume of the bulb and the capillary tube up to the zero mark of the stem, at this temperature is therefore,

$$V_1 = [m(2.00106 - d) - l'v] \text{ cm}^3 \qquad (121)$$

The volume V_0, at 0° is readily found from this by applying the equation

$$V_0 = \frac{V_1}{1 + gt}$$

where g may be taken as 0.000025.

If it be desired to determine the coefficient of cubical expansion of the glass vessel more accurately, it is best to fill the dilatometer with pure mercury and determine the readings l'_1 and l'_0 at two different temperatures, say t° and 0° C. If M be the mass of the mercury, and the values of v' (equation 120) at the two temperatures v'_1 and v'_0, then

$$V_1 = Mv'_1 - l'_1 v \text{ and } V_0 = Mv'_0 - l'_0 v$$

from which g can easily be calculated.

(c) *Coefficient of expansion of a liquid.* Fill the dilatometer with the liquid under investigation and immerse it in a large glass vessel containing water. Vary the temperature of this bath and observe the resulting height of the liquid in the stem of the instrument. Be sure to leave the instrument in the bath long enough to insure thermal equilibrium between its contents and the bath. Stir constantly to keep the temperature of the bath uniform in all parts. Add small quantities of warm or cold water, if necessary, to keep the temperature of the bath constant.

Take four or five different temperatures, t_1, t_2, t_3,....., and observe the corresponding readings on the stem, l_1, l_2, ... Denote the related volumes by V_1, V_2,.....,then

$$V_1 = V_0 (1 + g t) + l_1 v_2 \text{ etc.} \qquad (122)$$

Plot volumes and temperatures, and calculate the mean coefficient of expansion from

$$\alpha_n = \frac{V_n - V_0}{V_0 t_n}, \qquad (123)$$

In case the expansion of water is to be studied in the neighborhood of 4° C, the bulb must be quite large or the capillary tube of smaller bore than in the average instrument, as the expansion of water at these temperatures is very slight. Plot the apparent and the real volumes as ordinates and the temperatures as abscissæ. Compute a table for the specific volumes, V/M, for the different temperatures.

FORM OF RECORD.

Exercise 39. To determine the coefficient of expansion of a liquid by means of the dilatometer.

Date.....

(*a*) Value of one scale division of stem.
Mass of mercury = ..., number of divisions = ..., v = ...

(*b*) Volume of bulb at 0° C.
Mass of dilatometer empty....., Temperature.....
Mass of dilatometer filled....., m = ...,
V = ...,
V_0 = ...,
Coefficient of expansion of the glass.

Mass of dilatometer filled.....	Reading	t	V	g =..
Mass of dilatometer.....				

(*c*) Coefficient of expansion of.....

Reading	t	V	$\frac{V}{m}$	α_1 = ...
				α_2 = ...

86. **Air-Free Water.** Air-free water is used in many cases for finding the volume of glass instruments. Such water may be prepared by boiling distilled water for half an hour. Another method for freeing water from the absorbed air is shown in Fig. 42. The water is contained in a flask which is closed by a rubber stopper through which passes a capillary tube of fine bore, extending almost to the bottom of the vessel. A water aspirator connected to the side tube produces a partial vacuum in the flask and air is forced in through the capillary tube and rises through the water. This air, rising through the water under diminished pressure, produces a state of unstable equilibrium in the air previously absorbed in the water and this absorbed air can now be seen rising in the form of small bubbles. The process can be hastened by giving the flask from time to time a smart blow with the hand.

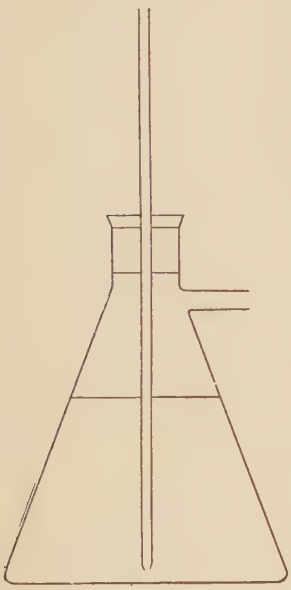

Fig. 42.

87. **Exercise 40. Constant Volume Air Thermometer.** If the volume of a given mass of gas be kept constant and its temperature be varied, the pressure of the gas upon the walls of its containing vessel will vary according to the formula

$$p_t = p_0(1 + \alpha t). \qquad (124)$$

where p_0 and p_t are the pressures at 0° and $t°$ C, respectively, and α is the pressure coefficient of the gas at constant volume. We know that for a perfect gas α is the same as the coefficient of voluminal expansion for the gas at constant pressure.[1] For all permanent gases the value of the pressure coefficient is nearly the same. In order to determine α, the gas, usually air, is kept at constant volume, its temperature is varied and the resulting pres-

[1] *College Physics,* Article 163.

sures observed. When properly calibrated the instrument furnishes the means of determining temperatures as a function of the observed pressures, and hence is termed a constant volume air thermometer.

The air is contained in the bulb *A*, Fig. 43, which is attached to a fine capillary tube *a*, bent twice at right angles, and joined at its outer end to a larger tube *B* about one centimeter in diameter.

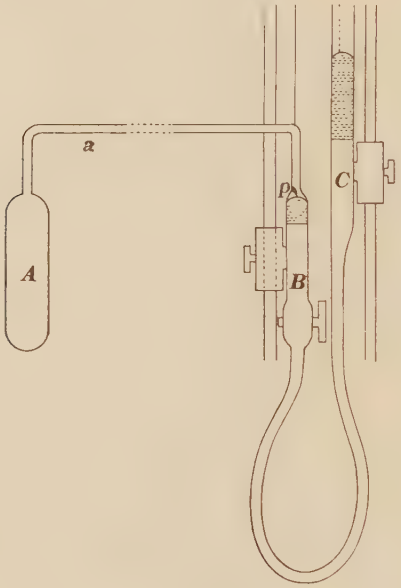

Fig. 43.

The tube *B* has at the point of junction with the capillary tube a fine pointer *p* of colored glass fused into its side, which serves as a point of reference for the height of the mercury column in *B*, and to which the mercury must be brought each time before a reading is taken. A mark on the capillary may serve the same purpose.

In the simpler forms of the apparatus the tube *B* is connected by a thick walled rubber tube to a second glass tube *C*, which is capable of considerable movement up and down a graduated scale placed between the two tubes. By means of this scale the level of

the mercury in each tube may be read off and the corresponding pressure at any temperature upon the gas determined. In this way the volume of the enclosed air is kept constant except for the change in volume of the bulb due to expansion, which must be allowed for in the final computation.

In order to determine the pressure coefficient of dry air at constant volume, the bulb is first carefully cleaned, dried, filled with dry air, and surrounded with melting ice. The tube C is then moved up or down until the surface of the mercury in B just touches the tip of the colored pointer. In this position the tube C is clamped and the reading on the surface of the mercury in each tube is noted. If the difference in these readings be h_0, and we call the barometric reading b, then the pressure upon the gas will correspond to a barometric height

$$H_0 = b + h_0. \tag{115}$$

The bulb is next surrounded by steam and the tube C adjusted so as to bring the mercury again to the tip of the pointer, and a second reading taken. Care must be taken to wait long enough before reading to allow the air in the bulb to reach the temperature of the steam, a condition clearly indicated by the level of the surfaces of the mercury columns remaining constant. If we denote the difference in level between the two columns by h, the air is under a pressure corresponding to a barometric height

$$H = b + h. \tag{126}$$

In measuring b and h it is sufficient to reduce both to the same temperature throughout the experiment, but for finding t, the temperature of the steam, it is necessary to reduce the barometric reading to 0° C, in order to make use of Table IX. In all manipulations of the instrument great care must be exercised to prevent any mercury from entering the capillary tube, and being drawn into the bulb.

From Gay Lussac's law[1] deduce the following formula for the pressure coefficient of a gas at constant volume where g denotes, as usual, the coefficient of voluminal expansion for the glass bulb:[2]

$$\alpha = \frac{H - H_0}{H_0 t} + g\frac{H}{H_0} \qquad (127)$$

What is the formula for any temperature t, as given by the air thermometer?

Insert the thermometer in water of different temperatures and compare the calculated temperatures with those measured by a mercury-in-glass thermometer. Plot temperatures and pressures.

In the derivation of the above formula it is assumed that the volume of the capillary connection from the bulb to the tip of the pointer is negligible as compared to the volume of the bulb. This assumption is the more nearly justified the larger the bulb and the finer the bore of the capillary tube.

FORM OF RECORD.

Exercise 40. To determine the pressure coefficient of dry air by means of the constant volume air thermometer.

Barometric pressure.....	Temperature	Reading		Pressure in cm of *Hg*
		B	*C*	
$\alpha =$				

CALORIMETRY.

88. Definitions. Temperature is to be sharply distinguished from **quantity of heat.** The former has reference only to the condition of a body affecting the sensation of warmth and cold and has no reference to the amount of matter involved. In **quantity of heat** account must be taken both of the **temperature**

[1] *College Physics*, Article 161.

[2] For the derivation of complete formula, see Kohlrausch, *Physical Measurements, 3d English from 7th German ed.*, pp. 93-97.

of the body and of its mass. The **unit of temperature** is the degree centigrade. The **unit of quantity of heat** is **the calorie.** A **calorie** is the quantity of heat required to raise the temperature of one gram of water 1°C. The **thermal capacity of a body** is numerically equal to the number of calories required to raise the temperature of **the body** one degree centigrade. The **thermal capacity of the substance,** of which the body is composed, is its **thermal capacity per unit mass,** or it is numerically equal to the heat in calories required to raise the temperature of one gram of the substance one degree centigrade. If we call the thermal capacity of a substance c, the mass of the body M, then the heat needed to raise the temperature of the body from t_2 to t_1 degrees is given by

$$H = cM(t_1 - t_2) \text{ calories} \qquad (128)$$

From the definition of the calorie it follows that the thermal capacity of water is taken as unity. Though c for water varies slightly with temperature[1], it will be considered as constant in the following exercises.

The **specific heat,** s, of a substance is the ratio of the thermal capacity of the substance to that of water, or

$$s = \frac{c}{c_w} \qquad (129)$$

Since c_w is unity the specific heat of a substance is numerically equal to its thermal capacity.

The specific heat of any substance is different for different temperatures and hence as usually given, it denotes the mean value for the specific heat between certain temperature limits.

89. Specific Heat by Method of Mixtures. If two substances of masses M_1 and M_2, at temperatures t_1 and t_2 and having thermal capacities c_1 and c_2, be brought into contact, they will come to some intermediate temperature t, such that the number of calories given out by the first is exactly equal to the number

[1] *College Physics,* Article 171.

gained by the second, provided of course, that no heat has been added from the outside, or been lost externally through conduction or radiation. Then the equation for the heat exchange is

$$M_1 c_1(t_1 - t) = M_2 c_2(t - t_2). \qquad (130)$$

If the second substance be water, then $M_2 = M_w$, $c_2 = c_w$ and the expression becomes

$$s = \frac{c_1}{c_w} = \frac{M_w(t - t_2)}{M_1(t_1 - t)}. \qquad (131)$$

In actual practice it is impossible to avoid loss of heat both by radiation and by conduction. If the water be contained in a vessel or calorimeter, then the latter receives heat along with the water and finally comes to the common temperature t. If M be the mass of the calorimeter and c its thermal capacity the amount of heat needed to raise its temperature from t_2 to t degrees is

$$H = cM(t - t_2)$$

This quantity must be added to the right hand member of equation (130). The calorimeter usually consists of a beaker, stirrer and thermometer for each of which cM must be calculated and their sum be taken in the above equation. If we set

$$m = \Sigma \frac{c}{c_w} M = \Sigma sM \qquad (132)$$

then m is the mass of water which has the same thermal effect as the calorimeter. This mass is called the **water-equivalent of the calorimeter.** Therefore the total heat gained by the water and calorimeter will be $c_w (m + M_w)(t - t_2)$ and we have

$$s = \frac{(m + M_w)(t - t_2)}{M_1(t_1 - t)} \qquad (133)$$

90. **Exercise 41. Water-Equivalent of a Calorimeter.** The calorimeter is a cup of thin metal, preferably of aluminium,

which is placed inside a large vessel upon a flat piece of cork or other poor conductor. In the calorimeter are a stirrer and a thermometer. Let m be the water-equivalent in grams of the calorimeter including stirrer and thermometer; also let the calorimeter contain M_w grams of water at a temperature t_2; suppose the resulting temperature, due to adding M_1 grams of water at t_1, finally comes to be t. Then the exchange of heat is represented by the equation

$$c_w(M_w + m)\ (t_2 - t) = c_w M_1\ (t - t_1) \qquad (134)$$

or, solving for m

$$m = \frac{M_1(t - t_1)}{t_2 - t} - M_w \qquad (135)$$

In practice weigh the calorimeter empty and dry, then fill about one-third full with water at a temperature about fifteen degrees above the temperature of the room and weigh again. The difference is M_w. Next add water at a temperature, about ten degrees below room temperature, until the resulting temperature after vigorous stirring is about room temperature. The temperature of the cold and warm water should be carefully determined, just before mixing, by means of a thermometer reading to tenths of a degree centigrade. The resulting temperature is to be taken only after the thermometer reading has become constant. Repeat the experiment twice, taking the mean of the three results as the water-equivalent. For the correction due to radiation see Art. 92.

FORM OF RECORD.

Exercise 41. To determine the water-equivalent of a calorimeter.

Date.....

Weight of vessel	Vessel with water 1st	M_w	t_2	t_1	t	Vessel with water 2d	M_1	m
.....	.....	...	....	....	....	.....	.....	....
.....	.....	....	....	.....	.....	.....	.....	.....

Compare this result with that obtained by multiplying the mass of the calorimeter by the specific heat of the metal, as given in Table X. In the case of a thermometer the exact masses of mer-

cury and glass are unknown. But the thermal capacities for equal volumes of mercury and glass are nearly the same, namely 0.47 calories per cc. To find the water-equivalent of a thermometer it is therefore sufficient to multiply the immersed volume of the thermometer by this number.

91. Exercise 42. Specific Heat of Copper. The piece of copper whose specific heat is to be determined is heated in a brass tube (Fig. 44), which is surrounded by a steam jacket. The copper is hung by a thread in the middle of the tube and the top is closed by means of a cork carrying a thermometer. The heater sits upon a wooden board having a hole of the diameter of the inner tube and directly beneath it. This board slides upon

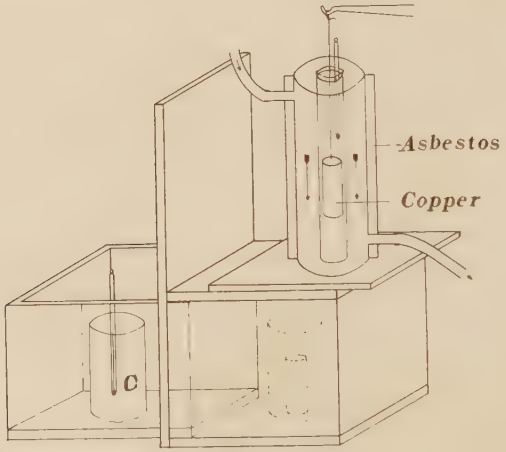

Fig. 44.

a support provided with a similar hole and so arranged that the two holes coincide when the board is pushed in as far as possible. Immediately under the hole in the support is placed the calorimeter so that the heated body may be passed directly from the tube through the support into the calorimeter. The sliding board is to shield the calorimeter from heat during the heating of the copper.

In use the dry copper is weighed and hung in place, the hole in

the support closed by the sliding board and steam passed through the jacket until the temperature of the interior becomes constant, t_1. This heating usually requires about twenty minutes. Meanwhile the calorimeter with the contained water is carefully weighed. The temperature of the water in the calorimeter is then read, t_2. The calorimeter is put in position, the sliding board pushed in, and the heated copper lowered gently into the vessel beneath. The calorimeter with its contents is then removed, the water thoroughly stirred and the highest temperature t, carefully noted. In order to avoid the necessity of correcting for radiation, it is well to have the temperature of the water in the calorimeter some 4° or 5° below the temperature of the room at the beginning the experiment. Apply formula (133).

FORM OF RECORD.

Exercise 42. To determine the specific heat of copper.

Date.....

Mass of Calorimeter	Calorimeter with water	M_1	M_w	m	t_1	t_2	t
.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....

Specific heat = ...

92. **Correction for Radiation.** In the preceding experiments the temperatures were so chosen as to render correction for radiation and absorption unnecessary. In experiments requiring a greater degree of accuracy this loss or gain of heat by the calorimeter must be taken into account. This is best done by noting times and temperatures for an interval of at least five minutes before the instant at which the experiment proper begins, that is, the instant at which the ice, steam or metal enters the calorimeter. Readings should be taken every twenty seconds. Plot the times as abscissae and the temperatures as ordinates, (Fig. 45). After the beginning of the experiment the temperature rapidly rises or falls, and having reached a maximum or a minimum it will practically become a linear function of the time for a few minutes, showing the rate of radiation or absorption.

According to Newton's law of cooling, the rate of loss or gain of heat energy by a body due to radiation, varies directly as the difference in temperature between the body and surrounding objects. From the straight parts of the curve determine this rate r, for a difference in temperature of one degree. Next determine the average temperature of the calorimeter for the time interval (*ab*, Fig. 45), during which the mixing occurs. This is done by dividing *ab* into a number of small intervals, taking the sum of the

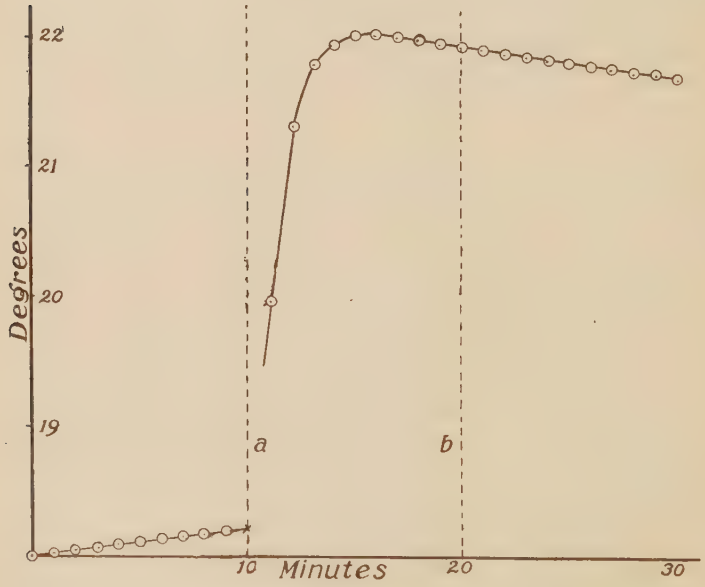

Fig. 45.

temperatures belonging to each, and dividing by the number of intervals. The difference between this average and room temperature multiplied by the rate r, gives the correction to be applied with its proper sign, to the observed reading at the time *b*.

A numerical example will make the method clearer.

The readings of the thermometer were begun ten minutes before

the heated body was introduced into the calorimeter, and the reading continued for twenty minutes after mixing took place. The room temperature was 19°.45 C.

Min.	Read.	Min.	Read	Aver. (Calc.)	Min.	Read.
0	18°.04	10	18°.19		20	21°.97
1	.06	11	19 .8	19 .0	21	.94
2	.07	12	21 .0	20 .4	22	.92
3	.09	13	21 .7	21 .4	23	.89
4	.10	14	21 .9	21 .8	24	.87
5	.12	15	22 .0	21 .95	25	.84
6	.13	16	22 .04	22 .02	26	.83
7	.15	17	22 .04	22 .04	27	.79
8	.16	18	22 .02	22 .03	28	.76
9	.17	19	22 .00	22 .01	29	.74
10	.....	20	21 .97	21 .98	30	.71

Average temperature,

Before mixing: 18°.12 C.
20th to 30th minute: 21°.83 C.

Rate of change of temperature per minute:

Before mixing: $\frac{18.17 - 18.04}{9} = 0.°0144$ C per minute.

20th to 30th minute: $\frac{21.97 - 21.71}{10} = 0°.026$ C per minute.

Rate per minute per degree,

Before mixing: $\frac{0.0144}{19.45 - 18.12} = 0°.011$ C per minute per degree.

20th to 30th minute: $\frac{0.026}{21.83 - 19.45} = 0°.011$ C per minute per degree.

Average temperature of the calorimeter from the 10th to 20th minute:

$$21°.46 \text{ C.}$$

Decrease in temperature due to radiation in 10 minutes,

$$10(21.46 - 19.45)0.011 = 0.°22 \text{ C.}$$

Highest temperature corrected for radiation:

$$21.97 + 0.22 = 22°.19 \text{ C.}$$

Total temperature change, corrected:

$$22.19 - 18.19 = 4°.00 \text{ C.}$$

93. **Exercise 43. Heat of Fusion of Water.** Water absorbs definite amounts of heat energy on passing from the solid to the liquid, and from the liquid to the gaseous state. The quantities of heat per gram thus absorbed are termed the **heat of fusion** and the **heat of vaporization.** The **number of calories necessary to change one gram of ice at o°C. to water at o°C.** is therefore numerically equal to **the heat of fusion of water.** This may be measured in various ways. One of the simplest is by the method of mixtures; *i. e.*, a known mass of ice at o°, is added to a definite mass of water at a known temperature, and the temperature of the water at the end of the melting enables us to compute the amount of heat consumed in melting the ice.

Thus let M_2 grams of ice at zero, be added to M_1 grams of water at t_1 and let the temperature at the end of the melting be t; also let the water-equivalent of the calorimeter, stirrer and thermometer be m, and let l denote the heat of fusion of ice as defined above. Then since the water formed by the melting of the ice must be warmed to t degrees, we have the heat lost by the calorimeter and its contents equal to the heat absorbed by the ice and the ice-water. Hence

$$M_2 l + c_w M_2 t = c_w (M_1 + m) (t_1 - t) \qquad (136)$$

from which since c_w equals unity, we have the numerical equality

$$l = \frac{(M_1 + m) (t_1 - t)}{M_2} - t. \qquad (137)$$

The apparatus consists of a calorimeter, a thermometer, and a circular stirrer covered with wire gauze to keep the pieces of ice under water while melting. Weigh the calorimeter, fill nearly full of water at a temperature t_1, about fifteen degrees above room temperature and weigh again. The difference is the mass of water M_1. Break clean ice into small pieces and add to the water sufficient dry ice to bring the temperature of the calorimeter and its contents to about fifteen degrees below room temperature,

when all the ice is melted. Stir vigorously throughout the operation, read the temperature t, as soon as the ice is all melted, and weigh the calorimeter and its contents once more. The difference between the last two weighings gives the mass of the ice M_2, that was added.

FORM OF RECORD.

Exercise 43. To determine the heat of fusion of water.

Weighings of calorimeter: Date.....

Alone	With M_1	With $M_1 + M_2$	M_1	M_2	m	t_1	t
.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....

Heat of fusion of water = ...

94. **Exercise 44. Heat of Vaporization of Water at Boiling Point.** The heat of vaporization at the boiling point is the number of calories per gram required to change water at that temperature into steam at the same temperature. Conversely if one gram of steam at this temperature be condensed into water the same number of calories will be liberated. Thus if M_2 grams of steam at a temperature t_2, having been conducted into a calorimeter of water equivalent m, containing M_1 grams of water at temperature t_1, produce by condensation and cooling a resultant temperature t, we may write our equation of heat thus:

$$M_2 L + c_w M_2 (t_2 - t) = c_w (M_1 + m) (t - t_1) \qquad (138)$$

or, since c_w equals unity

$$L = \frac{(M_1 + m) (t - t_1)}{M_2} - (t_2 - t). \qquad (139)$$

where L is the heat of vaporization of water. The water equivalent m, of the calorimeter, may be found experimentally as before, or by multiplying the mass of the calorimeter by its specific heat.

The determination may be made by either of the following methods:

First Method. Steam generated in a suitable flask is passed

through a wide tube 15 cms long and 3 cms wide, (Fig. 46), in which the water from condensation is caught and retained. From this it passes directly into the water in the calorimeter. The mass of steam condensed, M_2, is determined from the increase in weight of the calorimeter. The calorimeter having been carefully dried and weighed is filled nearly full of water at a temperature some fifteen or twenty degrees below that of the room and again weighed. The difference in weight is M_1. After the steam passes freely from the vertical tube leading from the water trap, the calorimeter and its contents are brought into place and the steam passed directly into the water until its temperature is some fifteen or twenty degrees above the temperature of the room. The water should be vigorously stirred during the condensation.

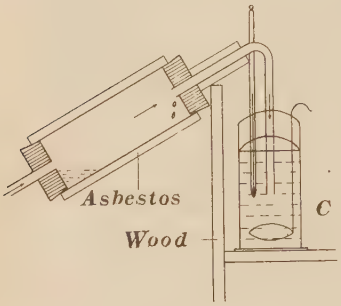

Fig. 46.

The calorimeter is then removed and the stirring continued until the temperature reaches a maximum, when the reading t_1 is taken and recorded. The temperature t_2, of the steam entering the calorimeter is to be determined by taking the barometric reading at the time of the experiment, and referring to Table IX. A third weighing determines the mass of steam condensed, M_2.

Since M_2 is usually a small mass any loss of water due to drops adhering to the exit tube from the water trap leads to a relatively large error in the mass of steam condensed, and should be taken into account for accurate work. If steam be allowed to enter the calorimeter too rapidly, the tube leading into it is covered on its inner surface with minute drops which are difficult to recover. Owing to the large influence of the above sources of error the following method is to be preferred.

Second Method. The apparatus (Fig. 47), consists of a closed copper vessel or calorimeter, provided with a stirrer and an opening for a thermometer; inside the closed vessel is a second smaller vessel into which steam is passed and there condensed. The

calorimeter proper having been carefully dried and weighed, is filled nearly full of water at a temperature some 15° to 20° below room temperature and again weighed. The difference is M_1. Steam is then generated in a small glass retort connected with the inner vessel into which the steam passes and condensing gives up its heat to the calorimeter and its contents. The temperature of the cold water t_1, is to be taken just before the steam enters the calorimeter.

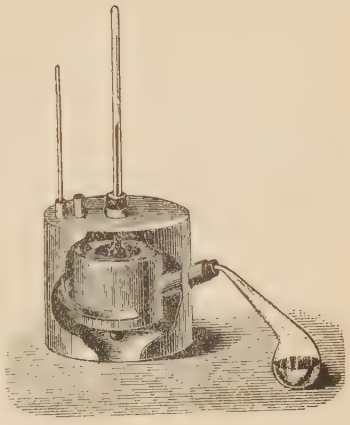

Fig. 47.

Steam is allowed to pass in until the resulting temperature rises as much above room temperature as the initial temperature of the water was below it. The flame is then removed and the temperature t, carefully determined. The amount of steam condensed is found by weighing the small retort before and after the experiment. The difference is the mass of steam condensed, M_2. The temperature t_2, of the steam entering the calorimeter is to be determined from the barometric reading as before. Care must be taken to prevent radiation from the retort and the flame under it from reaching the calorimeter. Stirring should be continued after removal of the flame until the temperature ceases to rise.

FORM OF RECORD.

Exercise 44. To determine the heat of vaporization of water.

Date.....

Weight of calorimeter = ...
Specific heat of calorimeter = ...
Water equivalent m = ...
Weight of calorimeter with water = ...
M_1 = ...
Weight of retort before experiment = ...
Weight of retort after experiment = ...
M_2 = ...

$M_1 + m$	t_1	M_2	t_2	t	L
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....

95. **Exercise 45. Melting Point and Heat of Fusion of Tin.** The tin is contained in an iron vessel *I*, Fig. 48, which is closed by a cover, provided with a slit to admit the stirrer *S*, and carrying a narrow tube *T*, to receive a thermometer reading to 360° C, or a thermoelement. The tube should extend well into the tin, and contain a small quantity of mercury to insure good thermal contact. In case a thermometer is used the stem correction (Article 81) must not be neglected. It is best to place the iron vessel inside a larger one, or surround it with an asbestos screen in order to avoid irregularities in cooling due to draughts of air.

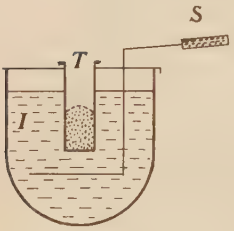

Fig. 48.

Heat the vessel slowly until all the tin is fused and the temperature has risen to about 280° C. Turn out the flame and take temperature readings every minute, stirring the molten metal

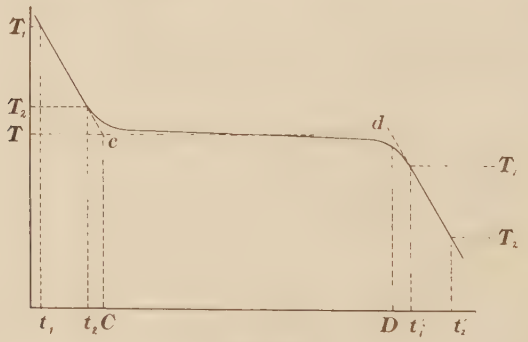

Fig. 49.

constantly until it solidifies. Continue the readings until the temperature has fallen to about 180° C. Plot times and corrected temperatures. The curve obtained will be similar to that shown in Fig. 49. A study of the curve thus found will show the following facts:

1. Solidification occurs where the curve becomes practically

horizontal. The mean temperature reading for this part of the curve is the melting point of tin.

2. To find the heat of fusion of tin, determine the rate of cooling before the melting point is reached and after the metal is all solidified. These rates are given by the expressions $\frac{T_1 - T_2}{t_2 - t_1}$ and $\frac{T_1' - T_2'}{t'_2 - t'_1}$ where T denotes temperature, and t denotes time.

If now we call the specific heat of molten tin $s_1 = 0.064$, and that of solid tin $s_2 = 0.060$, and if M represents the mass of tin, and m the water-equivalent of the iron vessel, stirrer, etc., then the quantity of heat Q_1, lost by radiation during the interval t_2—t_1, is, since c_w equals unity, given by the numerical equality

$$Q_1 = (M s_1 + m)\ (T_1 - T_2) \tag{140}$$

and that lost during the interval $t'_2 - t'_1$, is

$$Q_2 = (M s_2 + m)\ (T_1' - T_2') \tag{141}$$

and for R, the rate at which heat is lost we have the two values

$$\begin{aligned} R_1 &= (M s_1 + m)\ \frac{(T_1 - T_2)}{t_2 - t_1} \\ R_2 &= (M s_2 + m)\ \frac{(T_1' - T_2')}{t'_2 - t'_1} \end{aligned} \tag{142}$$

Take the mean of these two values for R.

Continue the straight parts of the curve where only cooling occurs, until they intersect the line of constant temperature in the points c and d. We may then assume that during the time $t = C D$, corresponding to the era of constant temperature $c d$, the process of solidification developed sufficient heat to supply the

loss due to radiation and thus maintained the temperature constant. This quantity of heat is ML, if L be the heat of fusion of tin. Again the quantity of heat radiated during this interval is Rt, hence

$$ML = Rt \tag{143}$$

and

$$L = \frac{Rt}{M}. \tag{144}$$

FORM OF RECORD.

Exercise 45. To determine the melting point and heat of fusion of tin.

Date.....

Weight of vessel = ...
Weight of tin = ...
Water equivalent = ...

Time	Temperature	$T_1 - T_2 = \ldots$ $T_1' - T_2' = \ldots$	$t_1 - t_1 = \ldots$ $t_2' - t_1' = \ldots$ $t = \ldots$	$R_1 = \ldots$ $R_2 = \ldots$ $R = \ldots$

Fusing point of tin = ... $L = \ldots.$

VAPOR PRESSURE.

96. Measurement of Vapor Tension. The **vapor tension** of a liquid is the pressure, measured in millimeters of mercury at 0° C, exerted by its saturated vapor produced by evaporation in a vacuum. An increase in temperature produces an increase in vapor tension, but vapor tension increases at a more rapid rate than the temperature. When the pressure of the saturated vapor above a liquid becomes equal to the atmospheric pressure exerted upon its surface, the liquid begins to boil. We are thus in position to measure vapor tension in two ways:

(*a*) *By measuring the depression of a barometric column of mercury, due to saturated vapor formed in a vacuum.*

(*b*) *By determining the boiling point under different pressures.* The curve showing the vapor tension as a function of the temperature, is called the vapor tension curve.

97. **Exercise 46. Vapor Tension of Ether.** The apparatus, Fig. 50, consists of a U-shaped tube with unequal arms, of which the shorter, about 50 cms in length, is closed and the other open. The short arm and part of the longer arm of the tube is first filled with mercury, the mercury being boiled in the tube to expel the air and then a small quantity of air free ether is introduced into the shorter arm.

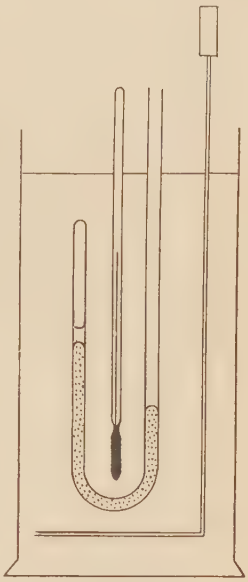

Fig. 50.

Insert the tube into a tall beaker filled with water and furnished with a thermometer and a stirrer. Vary the temperature of the water between 10° and 40° C, and measure the vapor pressure for each temperature with a cathetometer or with a meter stick. Care must be taken not to raise the temperature of the water at any time much above the normal boiling point of ether. Enough ether should be in the tube so that at any temperature obtained some of it remains in liquid form. Note the barometric reading. Plot the total pressure in cms of mercury as a function of temperature and determine from the curve the boiling point for a pressure of 760 mms of mercury.

FORM OF RECORD.

Exercise 46. To determine the vapor tension of ether.

Date.....

Temperature	Reading on short arm	On long arm	Difference	*P* in cms of *Hg*

Normal boiling point.....

98. **Exercise 47. Vapor Tension of Water at Various Temperatures.** As pointed out in the previous exercise, a liquid begins to boil as soon as its vapor tension equals the pressure upon the liquid. The easiest way therefore to determine the vapor tension of a liquid at temperatures near its boiling point, is to

vary the pressure upon the liquid and determine the temperature at which it will boil under the new pressure.

The apparatus Fig. 51, consists of a metallic flask *A*, containing the boiling liquid, and furnished with a tight fitting rubber stopper, through which pass a thermometer and the glass tube leading to the condenser *C*. A second flask *B* is connected to the upper end of the condenser and to a stopcock tube to which

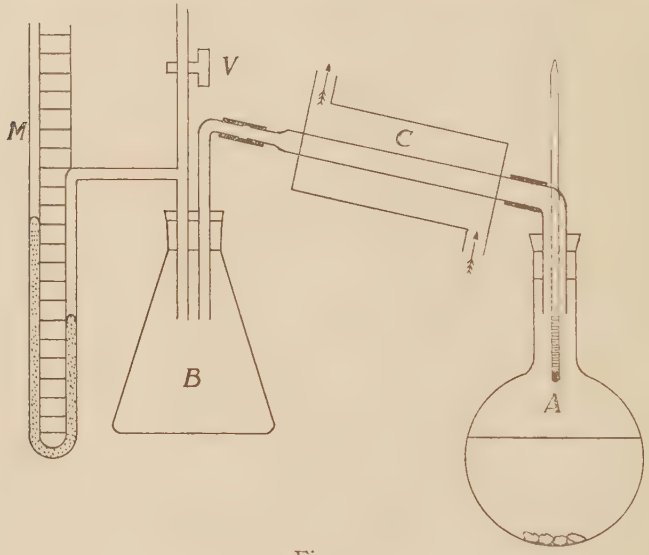

Fig. 51.

the small mercury manometer *M* is fused. This tube can be closed against the external air by the stopcock *V*. The reading of the barometer at the time of the experiment plus or minus the difference in the height of the mercury in the two arms of the manometer gives the pressure under which the liquid boils, and the thermometer gives the corresponding temperature.

A simple method of increasing the pressure above the liquid is to start with the stopcock *V* closed when the flame is placed under the flask *A*. Soon the pressure will rise. If necessary the pressure may be reduced by small steps by quickly turning the stopcock around 180 degrees.

In order to prevent radiation from the thermometer bulb towards the cooler outside it is advisable to surround it by a cylinder of asbestos.

Vary the pressure over ranges assigned by the instructor, and note the temperatures at which boiling takes place in each case. Put glass beads in the flask *A*, to avoid bumping. Plot pressures and temperatures.

FORM OF RECORD.

Exercise 47. To determine the vapor tension of water at temperatures in the neighborhood of the boiling point.

Date.....

Barometer reading....	Temperature	Manometer		h	$b \pm h$
		Tube a	Tube b		
Thermometer No.....					

CHAPTER VIII.

ELECTRICAL MEASUREMENTS.

UNITS AND STANDARDS.

99. Resistance. The practical **unit of resistance** is the **ohm.** It is represented by the resistance offered to an **unvarying cur-**

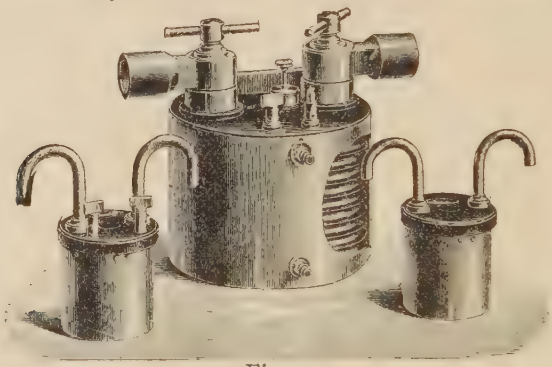

Fig. 52.

rent by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of length 106.3 centimeters.

Fig. 53.

For practical purposes standard ohms and multiples of the ohm are made of coils of wire, usually of **manganin,** an alloy whose resistance varies but little with the temperature, and which has a small thermo-electromotive force against copper, mounted in suitable protective cases. The makers usually furnish certificates for these coils issued by the testing laboratories of the different countries.

Fig. 52 represents some types of the German or Reichsanstalt standards. Their resistance varies slightly with humidity. This has led to the construction of standards in hermetically sealed

cases filled with oil which has been carefully freed from moisture and air. These standards were proposed by the Bureau of Standards in Washington and are called the N.B.S. standards of resistance, (Fig. 53).

For ordinary measurements, **resistance boxes** containing a number of coils of insulated wire, wound on bobbins **non-inductively,**[1] are in general use. The top of the box containing the coils is usually of ebonite and carries on its upper surface a number of heavy brass blocks so arranged that connection between adjacent blocks may be made by means of plugs inserted between them. The ends of the separate coils (Figs. 54 and 55), are fastened to the ends of adjacent blocks, so that when any plug is removed the current passes from one block to the next by passing through the connecting coil. In this way any resistance may be added by removing the proper plugs.

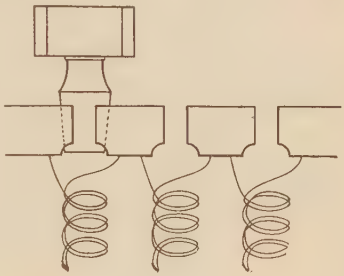

Fig. 54.

When the plug is inserted the current passes from block to block through the plug itself, the resistance of which must of course be negligible. On this account the plugs must fit accurately and be kept bright and clean, the ebonite must at all times be kept free from dust or moisture and **never be allowed to stand in the sun,** as the ebonite disintegrates slowly under the action of sunlight, a conducting layer of sulphur forms on the surface, and the efficiency of the coils is impaired. The plugs should at all times be handled by their hard rubber tops, they should be inserted with a gentle twisting motion, and **should all be loosened before the box is returned after use.**

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[1] Two methods of non-inductive winding are used; in the one case the required length of wire is measured off, doubled upon itself and then wound so doubled, upon the spool. In the second method the wire is wound in one direction only in each layer, but in opposite direction in consecutive layers. By this method the capacity of the coil, which by the first method may be quite appreciable, especially in coils of high resistance, is much reduced. (Chaperon.)

In the common form of resistance boxes there are only four coils for each decade, namely of 1, 2, 3 and 4 units, or 1, 1, 2 and 5 units (Fig. 86).

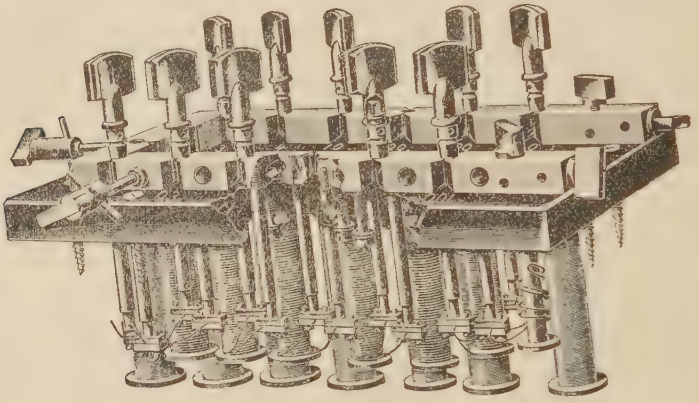

Fig. 55.

In the "decade" resistance boxes the inconvenience of adding up the resistances of the separate coils has been avoided, see Fig.

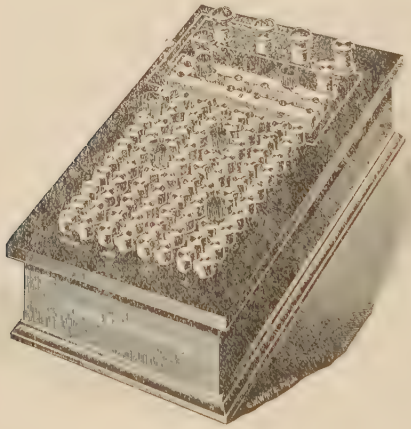

Fig. 56.

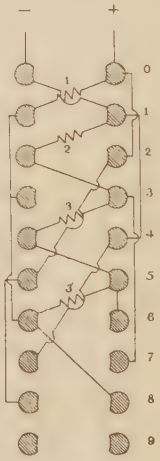

Fig. 57.

56. In each row of blocks representing a decade a plug is placed between two blocks and inserts in the circuit as many units as are indicated by the number on one of the blocks.

An ingenious method in which only four coils are used for each decade has been devised by Leeds and Northrup. The arrangement is shown by Fig. 57. In each decade there is one resistance coil of 1 unit, one of 2, and two coils of 3 units; for example if a plug be inserted between the two blocks, marked "4" the current must pass through resistances 1 and 3′, while 2 and 3 are shortcircuited. These boxes combine the advantage of the decade plan with cheapness of construction.

In the "dial" resistance boxes, built on the decade plan, the blocks are arranged in a circle and contact with a central ring is made by a sliding contact rotating around the center of the circle, (see Fig. 58).

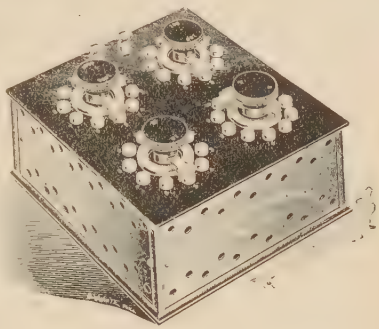

Fig. 58.

Previous to the adoption of the ohm various other units of resistance were employed and resistance boxes representing these earlier standards are sometimes met in practice. The most common are the **British Association Unit,** proposed by the British Association in 1864, and the **legal ohm** adopted at the Paris Congress in 1884. The relation between the ohm and these units is as follows:

1 ohm = 1.01358 B. A. units = 1.0028 legal ohms.
1 B. A. unit = 0.9866 ohm.
1 legal ohm = 0.9972 ohm.

In reporting work in measurements of resistance the student *should state explicitly which units have been used.*

The term **rheostat** is used for an unknown resistance of either fixed or variable value, and employed in work with currents exceeding 0.1 ampere.

The ends of the lead wires connecting the different instruments should be bright and clean and clamped firmly by the binding posts, since loose connections offer *high* and *variable* resistance. It is a bad practice to connect two wires by twisting them together.

100. **Current.** The practical **unit of current** is the **ampere.** The ampere is represented sufficiently well for practical purposes by "the unvarying current which will deposit silver from silver nitrate at the rate of 0.001118 grams per second."

101. **Electromotive force.** The practical **unit of electromotive force** is the **volt.** The volt is the electromotive force which steadily applied to a conductor whose resistance is **one ohm** will produce a current of one **ampere.** The **Cadmium** cell is chiefly used as the practical standard of electromotive force. This cell has for its positive electrode mercury covered by a paste of mercurous sulphate and a saturated solution of cadmium sulphate, and for the negative electrode cadmium amalgam, placed in the other leg of the H-shaped vessel, (Fig. 59), and covered by a saturated

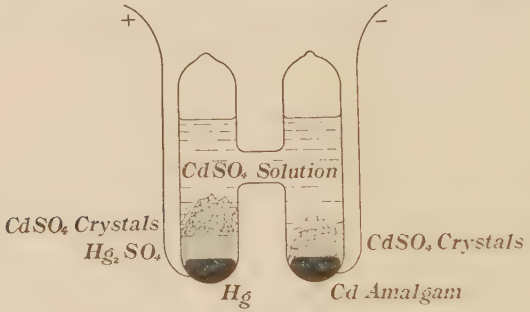

Fig. 59.

solution of cadmium sulphate. Saturation of the cadmium sulphate solution is secured by an excess of cadmium sulphate crystals. The glass tubes are hermetically sealed. The E. M. F. of this cell at the temperature t°C is given by the formula

$$E_t = 1.0183 - 0.00004\,(t - 20) \text{ volts.} \qquad (145)$$

The legal form of this cell is not portable and suffers the additional disadvantage of a slight lag in the E. M. F., since the density of the solution adjusts itself to a new temperature but slowly, some time being required for the cadmium sulphate crystals to dissolve or to crystallize out. This cell should never be placed on a short circuit. (Why?)

Some older forms of standard cells are still in frequent use.

The Clark cell, the first efficient form of standard cell, differs from the cadmium standard cell by the use of zinc and zinc sulphate in place of cadmium and cadmium sulphate. In other respects its construction is identical with that of the cadmium cell. The E. M. F. of the Clark cell is given by the formula

$$E_t = 1.433 - 0.00119\,(t - 15) \text{ volts.} \qquad (146)$$

The Carhart-Clark cell is an unsaturated form of Clark cell and its E. M. F. is about

$$E_t = 1.44 - 0.00056\,(t - 15) \text{ volts.} \qquad (147)$$

The Western Company manufactures portable unsaturated cadmium cells, which are extremely convenient on account of their negligible temperature coefficient. Their E. M. F. is very constant and about 1.019 volts; the exact value being readily determined by comparison with a primary standard.

Standard cells are used in all cases where it is necessary to determine the absolute value of an E. M. F. or of a potential difference. In many cases however, we wish simply to compare deflections of a galvanometer. In others the conditions may be so chosen that no current passes through the galvanometer. Such a method is termed a zero method. In experiments employing the zero method the Leclanché battery may be used. Where **steady deflections** are required however, a cell of **constant electromotive force** is necessary. In such cases we may use either a Daniell cell or a storage battery.

To set up a Daniell cell (Fig. 60), first fill the porous cup containing the amalgamated zinc, or negative electrode, two-thirds full of zinc sulphate solution and wait until the solution begins to moisten the outside of the cup, before placing the cup in the glass jar containing the copper, or positive electrode, and the copper sulphate solution. In order to avoid changes in the internal resistance during the experiment it is well to short circuit the cell for ten minutes before using.

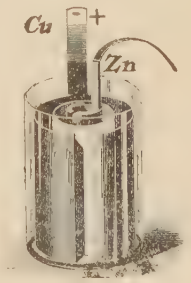

Fig. 60.

After use the cell must be taken apart, the zinc sulphate poured

back into its bottle, the porous cup thoroughly washed and *the zinc rubbed clean.* Copper oxide is usually deposited on the zinc as a black film. This may be readily rubbed off while it is moist, but if it be allowed to dry it adheres firmly and renders it difficult to amalgamate the zinc again. The E. M. F. of a Daniell cell is about 1.1 volts and this value may be used in all cases where great accuracy is not required.

When a constant E. M. F. of more than one volt is required, a *storage battery* ($E = 2.2$ volts) may be employed. On account of the very low internal resistance of a storage battery great care must be exercised to avoid short circuiting the cell. The student using a storage battery should *always leave one of the electrodes disconnected* until the instructor has seen and approved of the arrangement of the apparatus.

102. **Quantity of electricity.** The practical **unit of quantity** is the **coulomb.** The coulomb is the quantity of electricity transferred by a current of **one ampere** in **one second.**

103. **Capacity.** The practical **unit of capacity** is the **farad.** The farad is the capacity of a condenser which is charged to a **potential of one volt** by a **quantity** of **one coulomb.** The microfarad $= 10^{-6}$ farad, is commonly used as the measure of capacity. Secondary standards of capacity are made in the form of condensers with solid dielectrics. A large number of sheets of tin-foil interleaved with mica or paraffined paper are placed in a bath of melted paraffin in a vacuum chamber, to remove air bubbles, and allowed to cool. Alternate sheets of tin-foil are then connected and joined to one binding post, the remainder to another, thus forming the two ends or terminals of the condenser. A subdivided condenser is made by connecting all the leaves of one set to one bar, Earth, (Fig 61), as a terminal, and dividing the leaves of the other set into some number of divisions each of which is connected to a separate bar which in turn may be

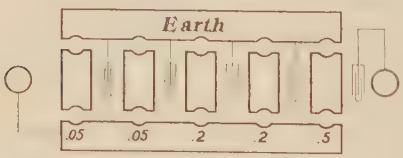

Fig. 61.

connected to a second binding post by means of a plug. When any division is to be used the plug is inserted for that division.

In another form, made by Leeds and Northrup, (Fig. 62), the subdivisions of the condenser are placed between two adjoining crossbars, thus allowing their use in series as well as in parallel. Where must the plugs be inserted to obtain 1 mf., 0.4 mf. or 0.25 mf?

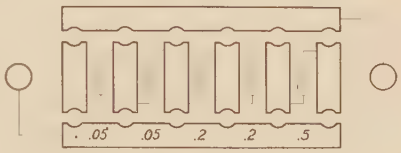

Fig. 62.

104. **Selfinductance.** The practical **unit of selfinductance** is the **henry**. A henry is the selfinductance in the circuit when the E. M. F. induced in the circuit is one volt, while the inducing current varies at the rate of one ampere per second.

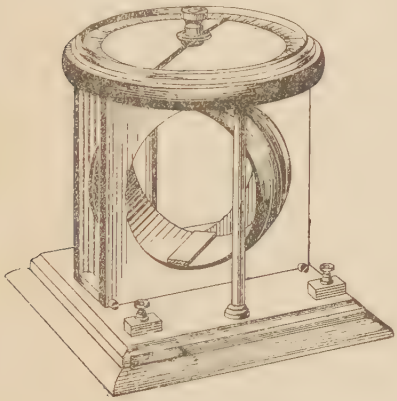
Fig. 63.

The usual form of selfinductance (Fig. 63)), consists of two coils in series, one fixed and the other movable about a diameter of the fixed coil as an axis. The movable coil may be rotated through 180° and in this way the selfinductance may be varied considerably. The instrument is calibrated empirically. The coils are wound on wood, and metal is, so far as possible, entirely avoided in the construction.

Other standards of inductance (Fig. 64), usually of single value only, are wound upon spools of nonmagnetic and nonconducting material, such as mahogany, marble or soapstone. The last is, however, generally magnetic in slight degree, and standards wound upon soapstone should be mistrusted.

Fig. 64.

INSTRUMENTS.

105. Keys. In most forms of apparatus for making electrical measurements, it is necessary to allow the current to pass through the measuring instrument for but a short space of time. Keys are provided for the purpose of closing and opening the circuit as may be desired. The ordinary key is so arranged that when pressed down, contact is made between two platinum points and the circuit is closed. On releasing the key the circuit is opened. For closed circuit work, plug keys or knife switches of various forms are used.

In work with the **Wheatstone's bridge** it is necessary to close the circuit through the galvanometer, **after** the current in the arms of the bridge has reached a constant value. For this purpose a double key (Fig. 65) is provided, in which the contacts for the circuit through the bridge arms, and that through the galvanometer, are made **successively** in the order mentioned. Such a key is termed a **successive contact key.**

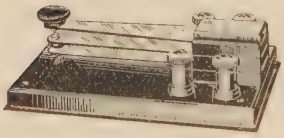

Fig. 65.

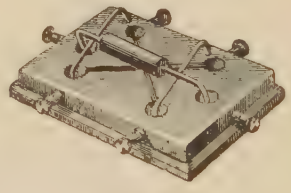

Fig. 66.

It is frequently necessary to reverse the direction of the current through an instrument without loss of time. This is most conveniently effected by means of the **Pohl's commutator** (Fig. 66). This consists of four cups containing mercury connected by cross wires as indicated in the figure, and a light frame of wires by which two other cups at the end of the block may be put in connection with the pair of cups on either side. If the source of current be connected to the pair of binding posts at the ends of the block, and the galvanometer to the two binding posts on either

side, the current is passed through the instrument in one direction or the other by tilting the frame from side to side.

All mercury contacts should be kept clean and should have the ends of the metal dipping into the mercury well amalgamated.

106. **Galvanometers.** The space about a magnet or about a wire carrying a current of electricity is called a **magnetic field.** Such a field is conceived to be filled with lines of magnetic induction. The direction of these lines is assumed to be the direction along which a free north seeking pole would tend to move. The lines of magnetic induction are said to run out from the north pole of a magnet, curve round through the air and re-enter the magnet at the south pole. In the case of a wire carrying an electrical current the lines of induction are concentric circles surrounding the wire.

Whenever two magnetic fields are brought near to each other there arises a stress between them, tending to turn the fields into such a position that they will mutually include the greatest number of lines of induction. Obviously the moment of this stress will be greatest when the two systems of lines of induction stand at right angles to each other. This is the position adopted in **galvanometers** and **electro-dynamometers.** In instruments designed to measure currents, at least one of the **magnetic fields** must arise from the current flowing through a coil of wire and hence must **vary as the strength of the current.**[1] The other magnetic field may be produced by a permanent magnet as in the galvanometer, or by a second coil carrying a current, as in the electro-dynamometer. One of the magnetic fields must be capable of rotation. Galvanometers may be divided into two classes:[2]

(*a*) *Galvanometers with stationary coil and movable system of magnets;* **needle type;** (Fig. 67).

[1] *College Physics,* Article 257.
[2] *College Physics,* Article 260 and 261.

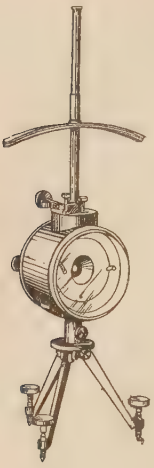

Fig. 67.

(*b*) *Galvanometers with stationary magnets and movable coil;* **d'Arsonval type.** (Fig. 68.)

In all measuring instruments the deflecting moment due to the current to be measured, must be balanced against a restoring or directing moment which tends to restore the system to its original position. When the system thus subjected to the action of two moments comes to rest we know that the moments of the deflecting and restoring forces are equal and opposite in direction. The angle of deflection is determined in one of several ways and the current determined as a function of this angle. The directing moment may be due to the action of an independent magnetic field upon the movable magnetic needle, or to the torsional moment of the suspending wire.

The **sensitiveness** of a galvanometer may be increased by decreasing the directing or restoring moment.

107. **The Astatic Galvanometer.** In the **astatic galvanometer** the moving system is composed of two magnets or systems of magnets of nearly equal strength placed one above the other with poles opposed. One of the needles is placed within a coil, the other needle either outside, or, better still, enclosed in a second coil through which the current flows in the opposite direction. In this way the magnetic moment of the system with respect to the earth's field is greatly reduced.

On the other hand a magnet placed under or over a movable magnetic needle may be made to exert any desired directive moment. Such a magnet is termed a **controlling magnet.** Some very sensitive galvanometers employ both an astatic system and a controlling magnet, the latter being placed in such a direction as to oppose the magnetic field of the earth.

108. **The d'Arsonval Galvanometer.** The d'Arsonval galvanometer, (Fig. 68), is more convenient for ordinary measurements. The coil (Fig. 69) is suspended by means of a fine metal wire or ribbon, and is attached to the base of the instru-

ment by another wire or metallic spiral spring. The current enters and leaves the coil by these upper and lower suspensions. The magnetic field of the permanent magnet is directed from N to S across the gap in which the coil is suspended. The current flowing through the coil sets up a magnetic field whose direction may be found by the right hand rule[1], and is indicated in the figure by the crosses $\times$, where the lines of induction enter the plane of the paper and by the dots $\cdot$, where the lines come out. The

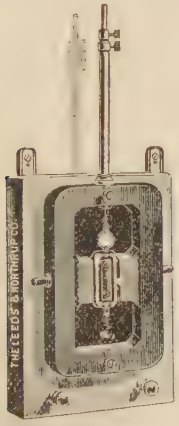

Fig. 68.

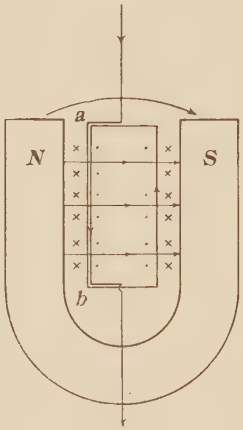

Fig. 69.

coil tends to place itself so that its lines of induction inside the coil are parallel to those of the field of the stationary magnet. Thus with the current flowing as indicated, the side *ab* tends to rise out of the paper. The coil comes to rest when the deflecting moment, due to the action of the fields, equals the torque in the suspension. The finer the suspension, the more sensitive is the instrument. Deflections are usually observed by means of mirror and scale. The d'Arsonval galvanometer has the great advantage that on closed circuit considerable **damping effect** is produced by **electro-magnetic induction** in the moving coil. *Why is this?* The degree of damping depends largely upon the resist-

[1] *College Physics*, Article 256.

ance of the circuit, being larger, the smaller the resistance. For each galvanometer a resistance of the circuit can be found, for which the swing of the coil changes from an **oscillatory** to an **aperiodic** motion. This limiting resistance is called the **critical resistance** of the circuit, and it can be shown that with this critical resistance the action of the galvanometer is quickest.

The instrument is also practically independent of the surrounding magnetic field and is consequently free from disturbances arising from variations in the intensity of this field, which often prove very troublesome in galvanometers of the needle type. The d'Arsonval galvanometer has the additional advantage that it may be **placed in any position,** while instruments of the needle type must be set so as to have the plane of the coils in the magnetic meridian. The needle type however, has usually greater sensitiveness, although the d'Arsonval type is sufficiently sensitive for most purposes.

109. **Methods of Observation.** In some instruments the moving system is furnished with a light pointer playing over a scale, but in the more sensitive galvanometers the deflections are observed by means of a mirror attached to the moving system. This mirror may be either concave or plane. In the first case an illuminated slit is focused by the mirror upon a semi-transparent scale and the deflections are read directly from the scale. In the second case a telescope is employed to view the image of a scale reflected in a plane mirror (see Article 29). Both methods are in common use.

110. **Shunts.** It frequently happens that the current to be measured will produce a deflection too great to be observed, or in some cases it may even endanger the instrument itself. In such cases we may reduce the current flowing through the galvanometer by means of a **resistance** connected **in parallel** with it. This resistance is called a **shunt.** Let g be the resistance of the galvanometer, s that of the shunt, then the resistance of the two

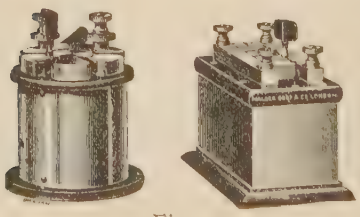

Fig. 70.

circuits in parallel is $\frac{g\,s}{g+s}$, and by Ohm's law,[1] if I denote the total current and I_g the current through the galvanometer, we have

$$I_g : I :: \frac{g\,s}{g+s} : g \qquad (148)$$

or

$$I_g = I\,\frac{s}{g+s}. \qquad (149)$$

If we observe the current I_g in the galvanometer, then the total current is $I_g.\frac{g+s}{s}$; where the factor $\frac{g+s}{s}$ is called the **multiplying power of the shunt.** Let the galvanometer resistance be n times that of the shunt, $(g = ns)$, then the multiplying power is $\frac{n+1}{1}$. If n be 9, 99 or 999, the corresponding values of the multiplying power are 10, 100, and 1000. The makers frequently furnish with the galvanometer shunt-boxes containing resistances equal to 1/9, 1/99, and 1/999 of that of the galvanometer, in which case the observed current I_g, is equal to 0.1, 0.01, or 0.001 I.

111. **Exercise 48. Calibration of a Galvanometer by Ohm's Law.** The object of this experiment is to determine whether the deflections of a galvanometer are proportional to the current flowing through it, or if that is not the case, to ascertain how the deflections vary with the current. To obtain currents which stand in a definite relation to each other we apply Ohm's law, by connecting the terminals of the galvanometer to different points of a circuit through which a **constant current** is flowing. Then the **potential difference**, *P. D.*, between the extremities of a resistance r, through which a current i is flowing, is equal to ir, and this potential difference causes the **current through** the **galvanometer** producing the **observed deflection.**

The simplest arrangement is to use a battery of constant E. M. F.[2] and to send the current through a straight wire, as for ex-

[1] It should be kept in mind that Ohm's law applies to constant currents only.

[2] The student may observe as a general rule, that a battery of constant E. M. F. is always to be used where *constant deflections* are to be obtained while ordinary cells may be employed for zero methods or ballistic methods.

ample, the wire of a slide wire bridge, (Fig. 71), and connect the galvanometer to two points, P and A, on this wire, where A denotes the position of the contact maker. Since the current through the wire must remain constant, no matter where the galvanometer is attached, it is evident that the galvanometer should have a very high resistance as compared with that of the wire. Explain this. The resistance of the wire of a slide wire bridge is usually about 0.2 of an ohm. If the galvanometer resistance is less than 2000 ohms, a resistance box R_2, with sufficient resistance to bring the resistance of the galvanometer up to 2000 ohms, should be put in series with it. The resistance box R_1 is inserted in the battery circuit in order to reduce the potential difference between P and A to a value such that with the maximum length of wire used in the experiment the deflections of the galvanometer will still be on the scale.

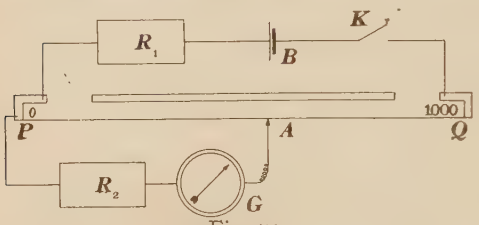

Fig. 71.

Move the point A, an ordinary contact maker, along the wire by steps of 5 cms, from 5 to 95 cms on the scale and observe the successive deflections of the galvanometer. Next reverse the battery current and repeat the observations. The mean of the deflections and the corresponding lengths between P and A are plotted. The **resulting curve** will be a **straight line**, if the deflections of the galvanometer are proportional to the current flowing through it. What principle besides Ohm's law has been applied in this experiment?

FORM OF RECORD.

Exercise 48. To calibrate a galvanometer by means of Ohm's law.

Galvanometer No..... Date.....
Resistance boxes R_1..... R_2.....

Length	Deflection a	Deflection b	Mean Deflection
.....	.....	.....	.....
.....	.....	.....	.....

112. **Exercise 49. Figure of Merit of a Galvanometer.** The figure of merit f, of a galvanometer is the ratio of the current

to the deflection which it produces and is measured by **that current** which will produce a **deflection of one scale division.** In case the scale is movable the distance of the scale from the mirror must be specified, or, still better, the result should be calculated for a distance of 100 cms. This exercise furnishes a simple application of Ohm's law.

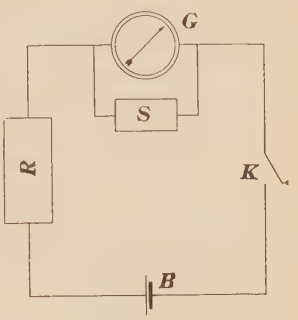

Fig. 72.

The exercise may be performed in either of the following ways:

(*a*) The arrangement is as shown in Fig. 72. Let B denote a battery of constant electromotive force E; R a very high resistance; g and b the resistances of the galvanometer and battery respectively; then

$$I = \frac{E}{R+g+b}, \tag{150}$$

if no shunt is needed, and

$$I = \frac{E}{R+\frac{gs}{g+s}+b} \tag{151}$$

if a shunt is used.

If the galvanometer show a deflection d, on the passage of a current I_g through it, we may assume, in the great majority of cases, that this deflection is proportional to the current, and we have the relation

$$fd = I \tag{152}$$

The factor f is the figure of merit of the galvanometer. Hence in the first case

$$f = \frac{I_g}{d} = \frac{I}{d} = \frac{E}{(R+g+b)d}, \tag{153}$$

and in the second case

$$f = \frac{I_g}{d} = \frac{I}{d} \cdot \frac{s}{g+s} = \frac{E}{\left(R+\frac{gs}{g+s}+b\right)d} \cdot \frac{s}{g+s} \tag{154}$$

Usually b as well as g, and still more $\frac{gs}{g+s}$, are negligible in comparison with R, and the formulæ become much simpler.

(*b*) For sensitive galvanometers which have no permanent shunt the following method is convenient: Let *B*, (Fig. 73), be a cell of constant electromotive force *E*, and close the circuit through *P* and *Q*. Then take the potential difference over *Q* to produce the deflection of the galvanometer. *Q* must be very small in comparison with *R*. Now the applied potential difference is $\frac{Q}{P+Q}E$ and our formula for *f* becomes

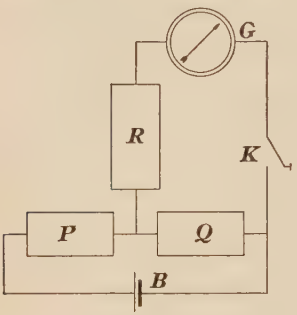

Fig. 73.

$$f = \frac{Q}{P+Q} \cdot \frac{E}{R+g} : \frac{1}{d} \qquad (155)$$

In practice vary the resistance *R* between 150000 and 250000 ohms and observe the deflections; in case (*b*) the ratio $\frac{Q}{P+Q}$ may also be varied.

The term sensitiveness of a galvanometer is frequently used in a different sense. It may be defined as the resistance which the circuit must have in order that one volt may produce unit deflection. This is termed the **ohm sensitiveness** of the galvanometer,

$$s = \frac{1}{f} \qquad (156)$$

The **potential difference per unit deflection** is called the **volt sensitiveness** of the galvanometer, a factor of the highest importance in most electrical measurements, as for example in work with the **Wheatstone bridge**, the **potentiometer** or the **thermal couple.**

If in Fig. 73 the electromotive force of the battery *E* be known, then the **volt sensitiveness** of the galvanometer can be shown directly to be

$$\frac{Q}{P+Q} \cdot \frac{E}{d} = = f(R+g). \qquad (157)$$

In order to give a definite meaning to this term, the resistance

in the galvanometer circuit (d'Arsonval galvanometer) should be the critical resistance. Many makers give the sensitiveness at the terminals of the galvanometer, but this is too indefinite since frequently, owing to **excessive damping**, the galvanometer cannot be used upon **short circuit.**

FORM OF RECORD.

Exercise 40. To determine the figure of merit of a galvanometer.

Galvanometer No..... $g =$ Date.....
Distance of mirror from scale

E	R	Q	P	$\frac{Q}{P+Q}$	d	f	$f(R+g)$
.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....

113. **Ballistic Galvanometers.** While in ordinary galvanometers it is required to observe deflections due to a steady current flowing through the instrument, or to prove the absence of a current from the absence of a deflection, it is often necessary to measure the quantity of electricity passing through the galvanometer, as in measuring the quantity of electricity stored in a condenser.

When a condenser is discharged through a galvanometer the current rises rapidly to a maximum, and then decreases to zero. In such a case a galvanometer having a coil with a large moment of inertia must be employed. Such an instrument is termed a ballistic galvanometer and its advantage consists in this, that the coil remains practically at rest until the entire quantity of electricity has passed through it. In this way the full force of the magnetic thrust is effective in starting the coil which moves off as if started by a blow. For small angular deflections the quantity of electricity may be set proportional to the deflection.[1] Here we observe the maximum deflection attained by the system on the first throw of the needle, and not a constant deflection. The quantity of

[1] Carhart and Patterson, *Electrical Measurements*, pp. 207-213.

electricity per unit deflection is called **the constant of the ballistic galvanometer,** or if the quantity Q gives a deflection d, then

$$c = \frac{Q}{d} \tag{158}$$

114. Constant of Ballistic Galvanometer. The **constant** of a **ballistic galvanometer,** especially in instruments of the d'Arsonval type, **depends** largely upon the **resistance** of the **galvanometer circuit.** In the following exercise it is determined with the galvanometer on *open circuit,* in which case it matters not how much resistance we add to the galvanometer. However the value thus found cannot be applied to the instrument when used upon a **closed circuit,** without taking into account the damping effect. This must be remembered in Exercises 74, 78 and 79.

115. Exercise 50. Determination of the Constant of a Ballistic Galvanometer. To determine the quantity Q giving a certain deflection d, use a battery of known electromotive force E, as a standard cell, and charge a condenser of known capacity by depressing the key to the point b (Fig. 74). Then by releasing the key, the condenser is disconnected from the battery and discharged through the galvanometer. Special keys, used for such experiments, known as charge and discharge keys, are shown in Fig. 75.

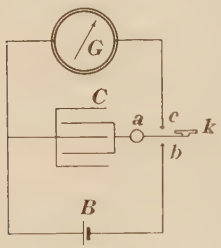

Fig. 74.

Letting c represent the constant of the ballistic galvanometer, Q the quantity discharged, and d the deflection, then

$$c = \frac{Q}{d} = \frac{EC}{d} \tag{159}$$

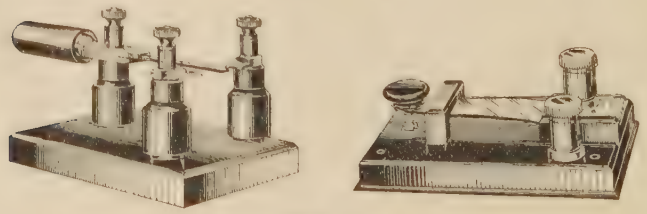

Fig. 75.

where C is the capacity of the condenser and E the E. M. F. of the standard cell. Change C by small steps and plot Q and d.

FORM OF RECORD.

Exercise 50. To determine the constant of a ballistic galvanometer.

Date.....

		E	C	Q	d	c
Galvanometer...	.....	.....	.....	.....	.....	.....
Condenser.....	Temperature...	.....	.....	.....	.....	.....
Standard cell....		.....	.....	.....	.....	.....

Mean = ...

116. **Voltmeters and Ammeters.** Voltmeters and ammeters are usually portable galvanometers of the d'Arsonval type. Wall instruments designed to measure high voltages or large currents, as in electric lighting or power stations, are usually attached to the switch board directly and give **continuous indication** as to the pressure and strength of the current furnished. Voltmeters and ammeters are direct-reading instruments, that is, they are so

Fig. 76.

Fig. 77.

calibrated as to show directly upon an arbitrary scale the **difference of potential** existing, or the **current flowing** between any two points to which they may be connected.

The best known instruments of this class are those designed by Weston. In these the directing couple is furnished by two spiral springs of **phosphor-bronze.** Rapid damping is secured by the use of aluminum frames upon which the coils are wound. The

voltmeter (Fig. 76) is commonly provided with two scales, one for high and one for low voltages. In the figure the low voltage terminal on the negative side is marked 15. Back of the needle is a mirror, and placing the eye in such a position that the image of the needle is hidden by the needle itself the error, due to parallax, in reading the scale is avoided. The Weston instruments are durable and remarkably accurate and the student should thoroughly familiarize himself with them before attempting to use them independently.

ELEMENTARY EXERCISES.

117. Exercise 51. Cells in Series and in Parallel. This simple exercise, designed to give the student some practice in the use of voltmeters and ammeters, consists in the study of the effect which a different grouping of cells has upon the current in an electric circuit. Use new dry cells for this exercise.

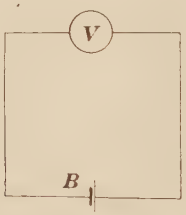

Fig. 78.

(*a*) Connect a voltmeter V to the terminals of a cell B, being careful to join the binding post of the voltmeter, marked +, to the positive pole (Fig. 78). Then replace the cell at B, first, by two cells in series and, second, by two cells in parallel. Take the readings in the three different cases and note the effect. Should the reading with the two cells in series be exactly twice that obtained with one cell?

(*b*) Arrange a circuit, consisting of a cell *B*, a rheostat *R* of about 10 ohms and a milammeter *A*, (Fig. 79). Again re-

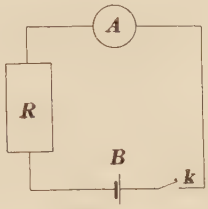

Fig. 79.

place the cell at *B*, first, by two cells in series and, second, by two cells in parallel. Note the effect of the different arrangements upon the reading and discuss the result.

FORM OF RECORD.

Exercise 51. Study the effect of different arrangements of cells.

Voltmeter No..... Date.....
Ammeter No.....

	Voltmeter reading	Ammeter reading
One cell	.....	.....
Two cells in series	.....	.....
Two cells in parallel	.....	.....

Discussion of results.

118. Exercise 52. Kirchhoff's Laws. Kirchhoff's laws[1] may be studied in the following manner:

(*a*) Arrange a battery *B* in series with three rheostats R_1, R_2 and R_3, (Fig. 80). Place an ammeter or milammeter successively in the positions *a*, *b*, *c* and *d*. Take the readings of the ammeter in each case. Does the current change in passing through a resistance?

(*b*) Arrange the rheostats in parallel (Fig. 81), and take the

[1] *College Physics*, Article 271.

readings of the ammeter, successively in positions *a, b, c* and *d*. Discuss the results as examples of Kirchhoff's first law:

$$\Sigma I = 0$$

(*c*) With the rheostats in series (Fig. 80) place a voltmeter

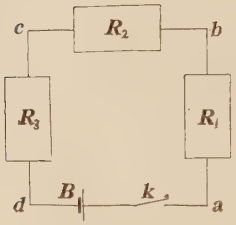

Fig. 80.

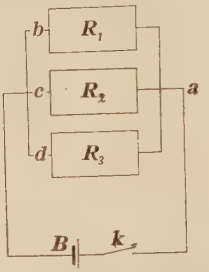

Fig. 81.

successively over *ab, bc, cd* and *ad* and read the differences of potential in each case.

(*d*) With the rheostats in parallel (Fig. 81) take the readings of the voltmeter over *ab, ac,* and *ad*. Discuss the results as examples of Kirchhoff's second law:

$$\Sigma I R = \Sigma E$$

It should be noted that an **ammeter** is always placed **in series** with the circuit and that a **voltmeter** is placed **as a shunt** between the two points whose potential difference is to be measured.

FORM OF RECORD.

Exercise 52. Kirchhoff's laws.

Date.....

Position of ammeter	Rheostats in series	Position of ammeter	Rheostats in parallel
a	.....	*a*	.....
b	.....	*b*	.....
c	.....	*c*	.....
d	.....	*d*	.....
Position of voltmeter		Position of voltmeter	
ab	.....	*ab*	.....
bc	.....	*ac*	.....
cd	.....	*ad*	.....
ad	.....		

Discussion of results.

119. Exercise 53. Resistances in Series and in Parallel. In order to prove the laws of resistance[1], a circuit is arranged consisting of a battery of two cells, a milammeter and three resistance boxes in series, with resistances $R_1 = 5$, $R_2 = 8$ and $R_3 = 20$ ohms respectively. Note the reading of the ammeter I. Take two boxes out of the circuit and adjust the resistance R in the remaining box until the ammeter reading I is the same or nearly the same as before. Be careful to have always some resistance in the circuit so as to **avoid shortcircuiting the cells.** The resistance R should agree with the equation

$$R = R_1 + R_2 + R_3 \qquad (160)$$

Next, arrange the three resistance boxes in parallel, making the resistance of each 15 ohms. Note the reading of the ammeter I_1. Remove one of the boxes and make the resistances of the remaining two boxes 15 and 10 ohms respectively. Note the reading of the ammeter, I_2. Finally leave only one box in the circuit and adjust its resistance R' until the current I'_1 is exactly or nearly equal to I_1; adjust again to R'' until the current is $I'_2 = I_2$. The results should agree with the equation

$$\frac{1}{R} = \Sigma \frac{1}{R_n} \qquad (161)$$

Give a sketch of the arrangement of apparatus for each of the different cases.

FORM OF RECORD.

Exercise 53. To study the laws of resistance.

Date.....

a. Resistances in series:

$R_1 =$	
$R_2 =$	$R =$ .....
$R_3 =$	
$I =$	$I' =$ .....

b. Resistances in parallel:

$R_1 =$			
$R_2 =$	$R' =$	$R_1 =$	$R'_1 =$ .....
$R_3 =$		$R_2 =$	
$I_1 =$	$I'_1 =$	$I_2 =$	$I'_2 =$ .....

[1] *College Physics,* Article 276 and 277.

MEASUREMENT OF RESISTANCE.

120. Exercise 54. Resistance by Substitution. The apparatus is arranged as shown in Fig. 82. B is a battery of constant E. M. F., x the unknown resistance, R a resistance box, G a galvanometer, K a key which may connect the galvanometer either to x or to R.

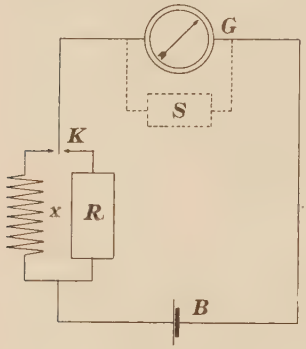

Fig. 82.

The battery circuit is first closed through the resistance x, and the galvanometer. If the deflection be too great, the controlling magnet or the torsion head may be so turned as to bring the deflection back upon the scale, or a shunt may be applied to the galvanometer. *Do not use a high resistance in the circuit instead of a shunt (Why?).*

Assuming that the deflections are porportional to the current we have

$$d_1 = c\,I_1 = c\,.\,\frac{E}{x+r}, \tag{162}$$

where r is the resistance of the whole circuit except x, and c is the proportionality factor. Next send the current through R instead of x; then

$$d_2 = c\,I_2 = c\,\frac{E}{R+r} \tag{163}$$

If now R be adjusted until the deflections in the two cases are equal, then

$$x = R. \tag{164}$$

If R cannot be adjusted so as to produce exactly the same deflection as x, interpolate between the two nearest values above and below. Measure three resistances separately and in series.

FORM OF RECORD.

Exercise 54. To measure three resistances by substitution.

Galvanometer.....	Temperature.....	Date.....
Resistance of.....	Resistance box.....	
Deflection with x	Deflection with R_1	Deflection with R_2
.....	.....	.....

$x_1 = \ldots\ldots\ldots\ldots$ ohms.

$x_2 = \ldots\ldots$ $\qquad x_1 + x_2 + x_3 = \ldots\ldots$

$x_3 = \ldots\ldots$

121. **Exercise 55. Resistance by Voltmeter and Ammeter.** This exercise consists in measuring **at the same time,** the **current flowing** through a wire and the **difference** of **potential** at its **terminal points.** Then if x be the resistance of the wire, we have by Ohm's law

$$x = \frac{V}{I}. \tag{165}$$

The potential difference at the terminals is measured by means of a voltmeter whose resistance must be very high in comparison with the resistance to be measured, as otherwise the current through the resistance x, will not be the total current I, measured by the ammeter. This is easily seen by applying the shunt rule,

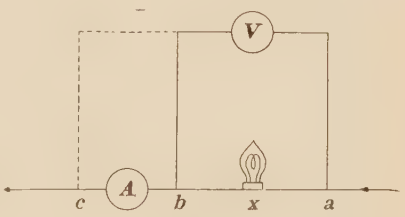

Fig. 83.

$$I_x = I \frac{R_v}{R_v + x}. \tag{166}$$

where R_v is the resistance of the voltmeter. If the resistance x is too large, it is better to include the ameter, whose resistance is very small, together with the resistance x between the terminals of the voltmeter, as shown by the dotted lines in Fig. 83, and then subtract the resistance of the ammeter from the resulting value of x.

Measure the resistance of an incandescent lamp. Put a rheostat in series with the lamp and the ammeter. Cut out resistance from the rheostat step by step until the lamp has reached its full candle power. Observe at each step the readings of the voltmeter and ammeter. Calculate the resistance for each current,

and the watts absorbed by the lamp; also the watts per candle when the lamp is burning at its normal voltage.

FORM OF RECORD.

Exercise 55. *To measure the resistance of an incandescent lamp by voltmeter and ammeter.*

Date .....
Voltmeter No..... Milammeter.....
Name of Lamp..... Candle power .....

V	*I*	*R*	*Watts*
.....	.....	.....	.....
.....	.....	.....	.....

Watts per candle = ...

122. **Exercise 56. Very High Resistance by Direct Deflection.** This method is a modification of the last. Instead of an ammeter, a galvanometer (Fig. 84) is used whose figure of merit is determined as in Exercise 49. The formula for the resistance then becomes

$$x = \frac{V}{f\,d} - g, \qquad (167)$$

since $I = f\,d$.

Usually g, the resistance of the galvanometer, is negligible in comparison with the high resistance X. High resistances of this kind are insulation resistances, as for example, the resistance offered to the passage of a current by the insulation of a cable or of a condenser. It is advisable to insert a high resistance *H.R.* in series with the galvanometer, in order to protect the instrument from excessive currents in case of a break in the insulation. If the deflections **vary notably with the time**, it is advisable to keep the key K closed and take a series of readings at definite time intervals. *If X represents the resistance of a condenser, short circuit the galvanometer by means of a shunt key, before closing the key K*, and afterwards open the short circuiting key to observe the steady deflection.

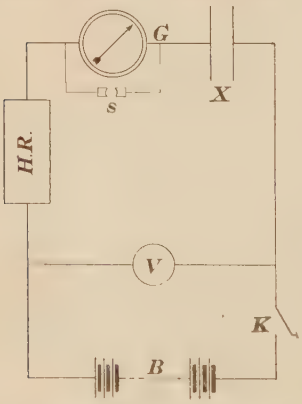

Fig. 84.

FORM OF RECORD.

Exercise 56. To measure the resistance offered by the insulation of a commercial condenser.

Condenser..... Date.....

Galvanometer	Time	V	d	x
Figure of merit, (Exercise 49)	.....	.....	.....	.....
Resistance of the galvanometer	.....	.....	.....	.....

123. **The Wheatstone Bridge.** The most accurate methods for measuring resistance are based upon the **Wheatstone bridge** principle. In its simplest form the Wheatstone bridge consists of a network of six conductors joining four points, *A, B, C, D,* (Fig. 85), so arranged that each point is joined to each of the other three points by separate conductors.

Let one of the conductors contain a source of E. M. F.; four of the others will form a divided circuit while the remaining one, containing a galvanometer, will form a **bridge** between the **two parallel conductors.** Let R_1, R_2, R_3, R_4 be the resistances forming the four branches of the divided circuit, and suppose them to be so adjusted that **no current** flows **through** the **galvanometer — Zero method.** Then it may be shown that the resistances satisfy the relation

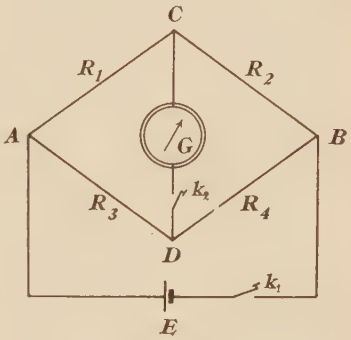

Fig. 85.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \qquad (168)$$

For let the potentials of the four points be represented by V_a, V_b, V_c, V_d, then since there is the same fall of potential between A and B, whether we pass by one route or the other, and since with a constant current the fall of potential is at all times proportional to the resistance passed over, we have

$$\frac{V_a - V_c}{V_a - V_b} = \frac{R_1}{R_1 + R_2} \text{ and } \frac{V_a - V_d}{V_a - V_b} = \frac{R_3}{R_3 + R_4}. \qquad (169)$$

But since no current passes through the galvanometer

$$V_a - V_c = V_a - V_d$$

therefore

$$\frac{R_1}{R_1 + R_2} = \frac{R_3}{R_3 + R_4} \qquad (170)$$

whence

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \qquad (171)$$

From the above relation it is evident that if three of the resistances be known the fourth may at once be determined. In fact it is necessary to know but one resistance and the ratio between the other two. Since the current through the parallel branches should become steady before the potential at C and D are tested, it is necessary to close the **galvanometer key last in every case.** A successive contact key is best adapted to this work. What would be the effect of **selfinductance** in one of the branches?

It will be found advantageous to follow Maxwell's rule:[1] "Of the **two resistances**—that of the battery and that of the galvanometer—connect the **greater resistance** so as to join the **two greatest** to the **two least** of the four other resistances." This insures the greatest sensitiveness of the apparatus.

Since both the potentials at C and at D depend directly upon the difference of potential between A and B, the absolute value of the latter or a slow change in the E. M. F. of the battery will not affect the balance of the galvanometer. A Leclanché cell may therefore be used as the source of current.

The **temperature** of the room must be carefully noted.

124. **Exercise 57. Resistance by Wheatstone Bridge Box.** A very convenient form of apparatus for the measurement of resistance by the Wheatstone bridge method is a box (Fig. 86), containing three known resistances connected in series, one of which may be given any value between one ohm and the maximum,

[1] Maxwell, *Electricity and Magnetism*, 3d edition, Vol. I, p. 478.

usually 10,000 ohms. The other two resistances form the proportional arms of the Wheatstone bridge and contain but a few coils, usually 1, 10, 100 and 1000 ohms. In this way the ratio between the known and the unknown resistances may be varied from 1000 to 0.001. The galvanometer is generally connected to the outer ends of the proportional coils and the battery to the remaining two binding posts. Frequently the two keys and a galvanometer are included in the case, thus making it a very compact and convenient instrument for the measurement of resistance. (Fig. 87.) This form of Wheatstone bridge is also known as the Postoffice Box.

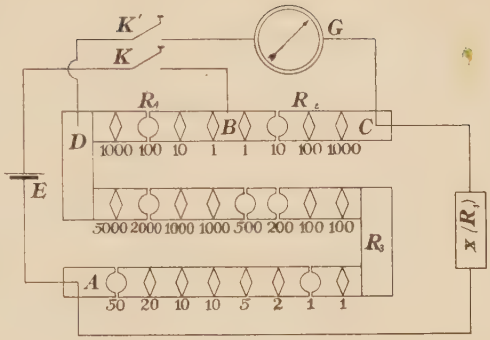

Fig. 86.

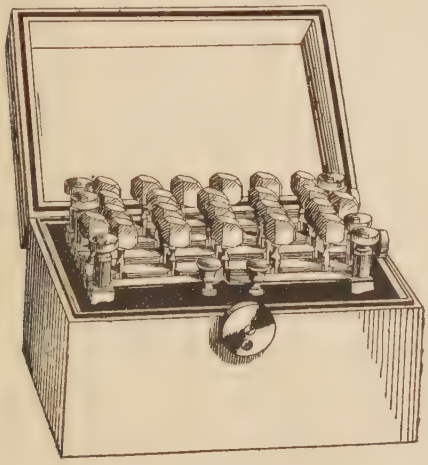

Fig. 87.

In practice, especially where the unknown resistance is not even approximately known, it is well to begin with equal resistances in the proportional arms R_2 and R_4. If the unknown resistance R_1 is too large the deflection on closing the galvanometer will be in one direction, if too small it will be in the opposite direction. First find two values for R_3 which change the direction of the deflection of the galvanometer, and **keep in mind the meaning of the direction of the deflection.** After having thus found the approximate value of the unknown resistance, change the ratio of the proportional arms so as to obtain the smallest value of the ratio R_2/R_4

that R_3 will allow and still produce a balance, and then make the final determination. Sometimes it may be necessary to interpolate between the values of R_3. Measure the resistance of three pieces of wire, and calculate the resistivity of each metal from the formula

$$\rho = R\,\frac{a}{l}\,, \qquad (172)$$

where R is the resistance, l the length and a the cross-sectional area of the wire.

FORM OF RECORD.

Exercise 57. To determine the resistivity of three metals.

Postoffice box No..... Date .....
Galvanometer .....
Temp. of room..... Temp. of wire.....

Specimen of wire	R_2	R_4	R_3	$R=R_1$	l	diam.	a	ρ
.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....

.....

125. Exercise 58. Resistance by Slide-Wire Bridge. From equation (171), it is seen that only the ratio R_3/R_4, and one resistance R_2, need be known, to effect the measurement of an unknown resistance. The ratio may be furnished by the two parts of a wire of uniform cross-section. In the slide-wire bridge the sum of the lengths of the two wires representing R_3 and R_4 is kept constant, usually 1000 mms, and the ratio of their lengths is changed by moving one of the galvanometer terminals along the wire. For this purpose a contact maker K' (Fig. 88), is substituted for the galvanometer key K_2 in the Wheatstone bridge. If on closing K and making contact at K', no current flows through the galvanometer, we have, neglecting the very low resistance of the copper straps,

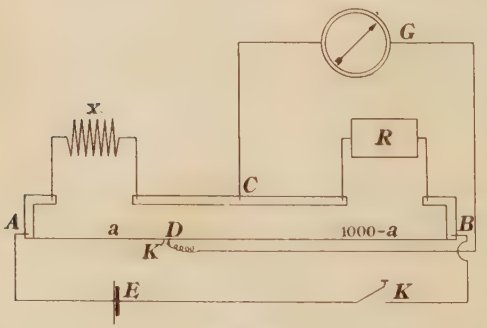

Fig. 88.

$$x = R\,.\,\frac{l_3}{l_4} \qquad (173)$$

or letting the reading at D on the wire equal a_1 mms, then

$$\frac{x}{R} = \frac{a_1}{1000 - a_1}. \qquad (174)$$

To obtain accurate results we must interchange x and R, in order to correct for possible errors due to **variations** in the **cross-section** of the wire, or to **unsymmetrical placing** of the **scale,** or to a **constant error** in the reading of the position of the contact maker. After exchanging x and R, a new reading, a_2, is obtained; then

$$\frac{x}{R} = \frac{1000 - a_2}{a_2}. \qquad (175)$$

or combining (174) and (175)

$$\frac{x}{R} = \frac{1000 + (a_1 - a_2)}{1000 - (a_1 - a_2)}. \qquad (176)$$

As shown on page 8, the errors of observation influence the result least when the contact maker is at the middle of the wire. That is when x and R are equal.

In some bridges four gaps are provided in the large copper strips for the insertion of resistance. In such bridges the two inner gaps are for the resistances of x and R, while the outer ones may be used to insert two nearly equal resistance coils which serve as extensions of the slide wire, thus enabling the experimenter to determine the ratio of the two lengths with greater accuracy. These additional coils must be determined in terms of mms of the bridge wire.[1]

Avoid moving the contact maker K′ while pressed down. A slight pressure should suffice to make contact. If not, the key and wire should be cleaned.

[1] Carhart and Patterson, *Electrical Measurements,* pp. 58-64.

FORM OF RECORD.

Exercise 58. To measure the resistance of an electromagnet. Study effect of self-inductance by closing K′ before K.

Slide-wire bridge No..... Date.....
Galvanometer..... Temperature.....

R	a_1	a_2	x
.....	.....	.....	.....
.....	.....	.....	.....

126. Exercise 59. Resistance of a Galvanometer.—Thomson's Method. The principle of the Wheatstone bridge may be applied to the measurement of the resistance of a galvanometer. Here the galvanometer is put in one of the four arms of the bridge and the battery circuit closed, producing a deflection of the galvanometer. If the deflection is too great, a resistance may be introduced into the battery circuit, to reduce the current—or the controlling magnet may be lowered or the torsion head twisted until the deflection comes once more upon the scale. Instead of a galvanometer as in the ordinary arrangement, a key is inserted, shown as K' in Fig. 89, or the contact maker may be used as before. If with a steady deflection of the galvanometer this key be closed and a current pass between C and D in either direction then the deflection of the galvanometer will change. Only when there is **no current through** K', will the scale **reading remain constant.** When a constant scale reading with K' open or closed is attained, the condition for a balance in the bridge is fulfilled. Then

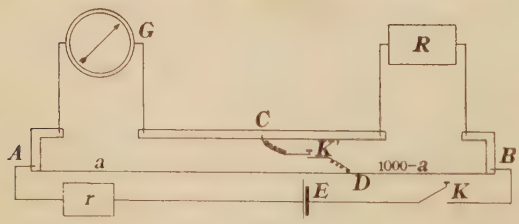

Fig. 89.

$$g = R\,\frac{a}{1000 - a} \text{ ohms.} \qquad (177)$$

In this case also the best results are obtained by making $R = g$. Exchange g and R and apply equation (176).

At first any resistance R_1, may be taken and the balance sought,

simply to find g approximately. Then make R nearly equal to g and repeat with greater care. Make three separate determinations by varying R. Must a cell of constant E. M. F. be used in this experiment?

FORM OF RECORD.

Exercise 59. To measure the resistance of a galvanometer.

Date.....

Temperature..... Galvanometer No.....

Preliminary	Final			
$R_1 =$	R	a_1	a_2	g
$a =$	.....	.....	.....	.....
$g =$	.....	.....	.....	.....

127. **Exercise 60. Resistance of a Galvanometer.—Second Method.** The resistance of a galvanometer may also be determined by the application of the principle involved in Exercise 48. The apparatus is arranged as shown in Fig. 71. With no resistance in R_2 the deflection of the galvanometer is observed for a length of bridge wire a_1, of about 30 cms. The length of the wire is next made about twice as large, that is 60 cms approximately, and sufficient resistance is added in R_2 to make the deflection of the galvanometer approximately equal to the first deflection. Finally move the contact key along the wire until the deflections are exactly equal. Call this reading of the bridge a_2. Then neglecting the resistance of the wire in comparison with that of the galvanometer

$$f d = \frac{a_1 I}{g} = \frac{a_2 I}{R_2 + g} \qquad (178)$$

$$g = \frac{a_1}{a_2 - a_1} R_2 \qquad (179)$$

Make three separate determinations of g.

FORM OF RECORD.

Exercise 60. To measure the resistance of a galvanometer by second method.

Date.....

Galvanometer No..... Temperature.....

a_1	a_2	R_2
		$g =$ .....

128. **Exercise 61. Resistance of an Electrolyte.** When a constant current is sent through an electrolyte, **polarization produces a counter E. M. F.** and so renders the principle of Wheatstone's bridge inapplicable. **Polarization** may be **avoided** by the use of **rapidly alternating currents.** The simplest source of such alternating currents is a small **induction coil** of high frequency. The galvanometer must be replaced by an instrument capable of detecting alternating currents. A telephone receiver of from 10 to 30 ohms resistance is best adapted for this purpose. Otherwise the manipulation is similar to that described in article 125. Vary the resistance or move the contact maker until the sound in the telephone ceases or becomes a minimum.

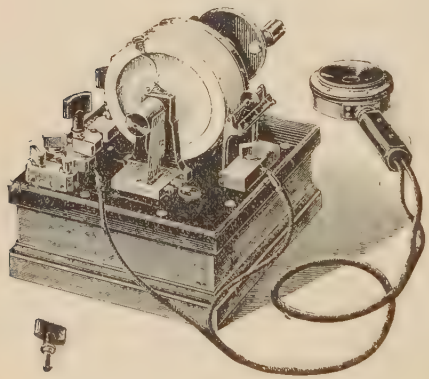

Fig. 90.

The most convenient form of bridge is one which has the wire wound on an insulating cylinder and in which contact is made by a small grooved wheel rolling upon the wire as the cylinder is revolved. This form of bridge, (Fig. 90), is due to Kohlrausch.

It is evident that self-induction must be avoided as far as possible in all parts of the apparatus. The electrolyte is con-

tained in a glass vessel furnished with two large platinum electrodes covered with **platinum black.** If the vessel has a cylindrical or prismatic form (Fig. 91) the resistivity may be readily determined from the formula given in Exercise 57. In electrolytes, however, the result is usually expressed in terms of the conductivity

Fig. 91.

$$k = \frac{1}{\rho} = \frac{l}{aR}.$$

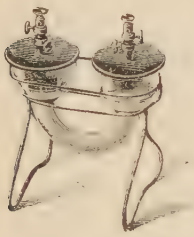

Fig. 92.

If the form of the vessel be irregular (Fig. 92), comparison must be made with an electrolyte of known conductivity k. As such a standard we may take a saturated solution of NaCl, sp. gr. 1,201 at 18° C. For such a solution

$$k = 0.216\,[1 + 0.023\,(t\text{-}18)]\ \text{ohm}^{-1}\ \text{cm}^{-1}.$$

See also Table XV for other solutions which may be used as standards. Letting the resistance of a standard NaCl solution be R and that of an electrolyte R_x, then $\frac{R}{R_x} = \frac{k_x}{k}$ and k_x the conductivity sought is given by the relation

$$k_x = k \cdot \frac{R}{R_x}. \qquad (180)$$

The quantity Rk is a constant for each vessel and is called the **resistance capacity** of the vessel.

Since the temperature coefficient of electrolytes is very large, great care should be taken in the determination of the temperature.

Frequently, especially in German tables, the conductivity is referred to that of mercury at 0°C. Since the resistivity of mercury at 0°C is 1/10630 ohm cm, its **conductivity** is 10630 ohm^{-1} cm^{-1}. To find k in the terms of mercury at 0°C divide the value found from (180) by 10630. The conductivity of electrolytes is usually expressed either as molecular conductivity or as equivalent conductivity.

The "**molecular conductivity**" μ is defined by the equation $\mu = k/m$ where m is the number of **gram molecules**[1] of the substance dissolved in one cubic centimeter of the solution.

The molecular concentration is, however, generally expressed as the number N of gram molecules, dissolved in a liter of the solution, so that N equals 1000 m. From this follows

$$\mu = \frac{1000}{N} k \qquad (181)$$

1000/N is the number of cubic centimeters containing one gram molecule of the dissolved substance. The conductance of a cubic centimeter of a body between two opposite faces of the cube is numerically equal to the conductivity of the substance (equation 172). If, therefore, that quantity of a solution which contains one gram molecule of the dissolved substance be brought between two electrodes, one centimeter apart and forming the sides of the containing vessel, the conductance observed will be numerically equal to the molecular conductivity of the electrolyte.

The **equivalent conductivity** λ, is found from the molecular conductivity by dividing the latter by the valence of the ions, *i. e.*, in case of acids by their basicidity, in bases by their acidity, in salts by the number of acid or basic valences present. Equivalent conductivity is a quantity of great importance in the modern electrolytic theory.

FORM OF RECORD.

Exercise 61. To determine the specific and molecular conductivities of a solution of a salt, using three different dilutions.

Date.....

	Standard:				Final measurements						
$t°$	a_1	a_2	r_2	R	Solution	a_1	a_2	r_2	R_x	k_x	μ_x
					$m =$						
					$m =$						
		$kR =$			$m =$						

[1] A gram molecule of a substance is a mass of the substance, in grams, equal to the molecular weight of the substance.

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE.

129. **The Voltmeter.** The simplest method for measuring a difference of potential between two points is to connect them to a voltmeter, or to any instrument whose graduation allows us to read off the potential difference in volts at its terminals. The voltmeter was employed for this purpose in Exercises 51, 52 and 55. The student should bear in mind that the **voltmeter** must at all times possess a very **high resistance** R_v, if its indications are to be reliable. Thus in measuring the drop in potential over a known resistance R, the instrument actually gives not $E = IR$, but $E' = I \cdot \frac{R \cdot R_v}{R + R_v}$. Now it is clear that E' can equal E only when R_v is so large that R may be neglected in comparison.

130. **Exercise 62. Electromotive Force of a Cell.** The apparatus (Fig. 93), consists of a battery B, of E. M. F. higher than that of the cell to be measured, a resistance R_1, and a voltmeter. The cell B', whose E. M. F. is to be determined, is joined in parallel with the voltmeter, so that the E. M. F. is in the same sense as that of the cell B. Close key K' and adjust the resistance R_1, until on closing key K, the reading of the voltmeter does not change. The current from B produces a difference of potential IR_v at the terminals of the voltmeter, where R_v is the resistance of the voltmeter. If this P. D. be smaller than the E. M. F. of the cell B', then upon closing K an additional current, taken from this cell, will flow through the voltmeter and increase the reading. If IR_v be larger than the E. M. F. of B' a part of the current I will be shunted through B' and the reading of the voltmeter be decreased upon closing K. Only if IR_v equals the E. M. F. of B' will no current flow through the cell upon closing K; and the voltmeter reading which is IR_v measures then directly the E. M. F. of B' no matter how large the resistance of the voltmeter is.

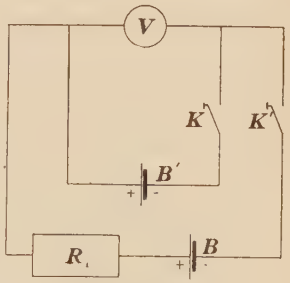

Fig. 93.

To increase the accuracy of the method a **galvanometer more sensitive** than the voltmeter may be inserted in series with B' and R_1 adjusted until the galvanometer shows no deflection on closing K. A high resistance in series with B', may be used at first to prevent the passage of too large a current through the cell, but it should be removed before making the final adjustment.

By placing one or more standard cells at B' this method is admirably suited for the calibration of voltmeters at points corresponding to the electromotive forces inserted.

FORM OF RECORD.

Exercise 62. To measure the E. M. F. of five different cells, first separately and afterward all joined in series.

Voltmeter No..... Date.....

Name of cell..... Voltmeter reading.....

131. **Exercise 63. Electromotive Force by Potentiometer Method.** In measurements of E. M. F. or of potential difference where a high degree of accuracy is required, it is necessary to compare the E. M. F. to be measured with that of a standard cell.

The arrangement of the apparatus is shown in Fig. 94. The principle of the method consists in producing by means of a current from the battery B, a potential difference at the terminals of a resistance R, equal to the E. M. F. of the cell B' whose electromotive force E is to be measured. The two electromotive forces should be so arranged as to oppose each other in the galvanometer branch. If now the keys K and K' be closed, no cur-

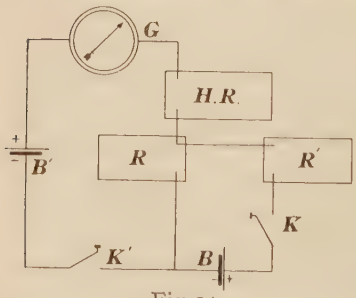

Fig 94.

rent will flow through the galvanometer when E equals iR. In practice a standard cell is first placed at B'. Let its known E. M. F. be E_1; then after the resistance R has been so adjusted that no current flows through the galvanometer on closing the keys, we have

$$E_1 = i_1 R_1 \tag{182}$$

The cell under experiment is next set in at B' and after a new adjustment of resistance for no current, we have

$$E_2 = i_2 R_2 \tag{183}$$

and under the condition that $i_1 = i_2$, we have at once

$$E_2 = E_1 \cdot \frac{R_2}{R_1} \cdot \tag{184}$$

The condition $i_1 = i_2$ is fulfilled when the battery B has a constant E. M. F., **and when the sum of the resistances R and R' remains constant throughout the experiment.** This sum should be at least 2000 ohms. It is moreover evident that the E. M. F. of B must be greater than that of B'. A pair of Daniell cells or a storage battery is best for this purpose. In this latter case the auxiliary circuit should be closed for about half an hour before beginning the measurements and be kept closed throughout the experiment. If the E. M. F. of ordinary open circuit cells is to be measured, a freshly charged Leclanché element will do very well as the high resistance of $R + R'$ prevents it from polarizing appreciably, if the key K is kept closed for but an instant. The high resistance $H.R.$ is inserted in the galvanometer circuit to prevent the passage of large currents through the instrument. It should be cut out when a balance is nearly obtained.

It is most convenient to use two boxes of the same kind in R and R' and keep the sum $R + R'$ equal to the sum of the resistances in one box. In this case it is easy to check the constancy of the sum, since, whenever a plug is removed in one box it has to be inserted in the corresponding hole of the other.

It is clear that a wire of constant length and uniform crosssection may be substituted for the two resistances R and R', and a balance obtained by shifting one terminal of the galvanometer along the wire. If the readings are a_1 and a_2 in the two cases, then

$$E_2 = E_1 \cdot \frac{a_2}{a_1}. \qquad (185)$$

In this case however, care must be taken to avoid heating the wire, and to this end the key K should be kept closed but an instant at a time.

FORM OF RECORD.

Exercise 63. To measure the E. M. F. of five cells, separately and in series.

Date.....

	Name of cell	R	R'	E
Galvanometer				
Resistance boxes				
Standard cell No.				
Temperature				
E. M. F. of standard cell.				

132. **The Potentiometer.** The frequent application of the **principle** of the **potentiometer** to the measurement of the electromotive forces of cells, of **thermo-couples,** or of the **potential difference** at the **terminals** of a **standard resistance** due to the passage of a current, has led to the construction of resistance boxes of special design called **potentiometers.** The principle is the same as that given above and a description of one of the many forms on the market will explain the method of using such an instrument.

A standard cell is connected through two binding posts N (Fig. 95) to the dial switches, which allow the galvanometer circuit to be connected to a **definite part** of the **total constant resistance** of the potentiometer. The switches must be so arranged that this portion of the **resistance** in the galvanometer circuit reads to some **decimal mul-**

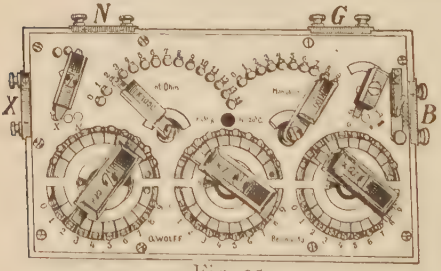

Fig. 95.

tiple of the **electromotive force** of the **standard cell.** The **auxliary current,** entering the instrument at *B,* is then adjusted by means of an external resistance, until a balance of the galvanometer is obtained. By the turning of the switch in the upper left hand corner, the **unknown potential difference** at *X* is substituted for the standard cell and the dial switches again adjusted until a balance is once more attained. The **potential difference** can then be **read off directly** from the **resistance indicated by the dials.** In some instruments the standard cell is connected to a separate

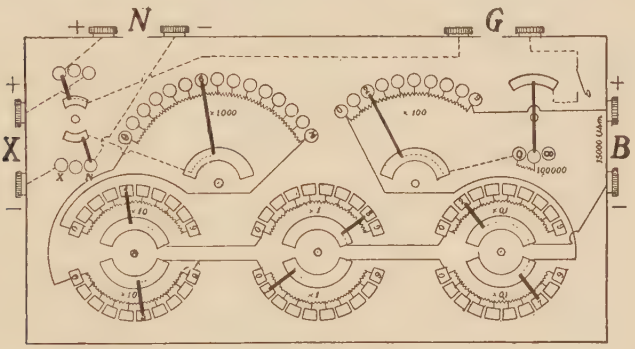

Fig. 96.

resistance outside the potentiometer proper, so that at any time the balance with the standard cell may be checked without disturbing the position of the dials.

133. **Exercise 64. Calibration of a Voltmeter.** This method also consists in balancing the E. M. F. of one or more standard cells against the potential difference at the terminals of a resistance traversed by a current. Let V be the difference of potential at the terminals of the voltmeter, E be the E. M. F. of the standard cells, and i the current flowing through R and R'; then

$$V = i\,(R + R') \text{ and } E = i\,R, \tag{186}$$

or

$$V = E\,.\,\frac{R + R'}{R}\,. \tag{187}$$

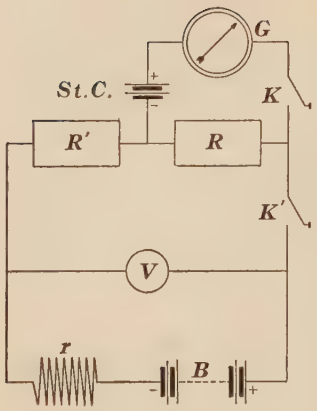

Fig. 97.

The arrangement is that shown in Fig. 97. The resistance r is introduced to vary the readings of the voltmeter, but it should be noted that the voltmeter reading will then vary more or less with the change of R or R'. If the accuracy of any individual reading of the instrument is to be tested a repeated adjustment of r may be necessary. In this case it would be more convenient to adjust the number of cells in B so as to give, as nearly as possible, the desired reading or to keep the sum R plus R' constant, which in general is not necessary.

The voltmeter must be kept in the circuit all the time during the experiment. A **high resistance** in the **galvanometer branch** before the final balance is reached is also advisable here. The voltmeter correction is the amount to be added to the voltmeter reading to make it correspond to the computed value of V. For the calibration of the low readings of a voltmeter see also Exercise 62. Plot voltmeter readings and corrections.

FORM OF RECORD.

Exercise 64. To calibrate voltmeter No. .....

Date.....

	Reading of voltmeter	R	R'	V	Correction
Galvanometer					
Standard cell No.....	.....	.....	.....	.....	.....
Resistance boxes....	.....	.....	.....	.....	.....
Temperature	.....	.....	.....	.....	.....

134. **Exercise 65. Thermoelectromotive Force of a Thermoelement.** A **thermoelement**[1] is usually used in such a way as to keep **one** of the **junctions** at a **constant temperature,** preferably 0°C., while the other junction is brought into contact with the point whose temperature is to be determined. If immersed in a liquid the junctions, with the wires well insulated from each

[1] *College Physics,* Article 303.

other, should be surrounded by thin glass or porcelain tubes in order to avoid **disturbances** due to **voltaic action.** For better conduction of heat the tubes may be filled with pure paraffin oil or kerosene.

Insert one of the terminal tubes in ice water and place the other in a water bath to be heated. The temperatures should be determined by means of **accurately calibrated thermometers.** Connect the free terminals of the couple to a **low resistance galvanometer,** whose **volt sensitiveness** is known. Vary the temperature of the water bath by eight or ten steps, between 0° and 100°C., and plot a curve, using **temperatures** as **abscissae** and **electromotive forces** as **ordinates.** It is usually most convenient to use the critical resistance for the galvanometer circuit, but in case it is found necessary to increase the resistance of the circuit, then the **volt sensitiveness** of the instrument must be **determined** for the **particular resistance** of the **circuit** as used. For more accurate work a **potentiometer,** preferably one of **low resistance,** should be used. A convenient form of thermoelement for low temperature measurements is a copper-constantan couple which gives about 0.000017 volt for a difference of temperature of one degree.

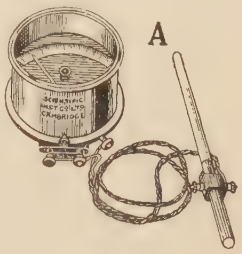

Fig. 98.

FORM OF RECORD.

Exercise 65. To determine the thermoelectromotive force of a thermoelement.

Date.....

Galvanometer	Temperature	Deflection	E. M. F.
Volt sensitiveness.....			
Resistance in circuit..			

ELECTROMOTIVE FORCE AND INTERNAL RESISTANCE OF BATTERIES.

135. **Electromotive Force and Potential Difference.** Whenever a battery is closed through a conductor and no external work is done except that of heating the conductor, we have by Kirchhoff's second law

$$E = I\,(R + r), \qquad (188)$$

where E is the electromotive force of the battery, I the current, R the external resistance and r the internal resistance of the battery. In case polarization occurs, the counter E. M. F. of polarization must be subtracted from E. According to the above formula the drop of potential in the whole circuit may be considered as divided into two parts: (a), $I\,R$, the drop of potential over the external resistance commonly called **the terminal potential difference,** and (b), $I\,r$, the drop in potential in the cell itself. The relative potentials of the various parts of the circuit may be represented as in Fig. 99, where the potentials are given as ordinates and the resistances as abscissae.

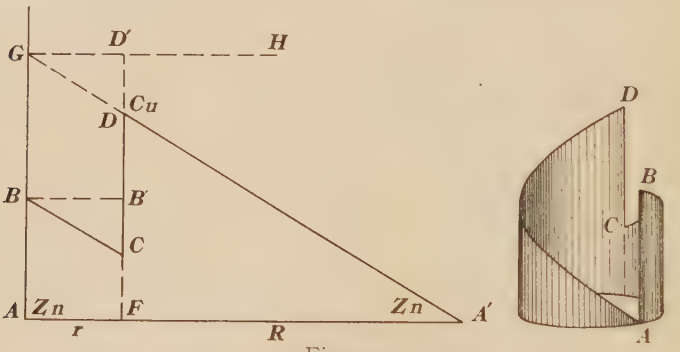

Fig. 99.

Consider the case of the Daniell cell. The potential of the zinc (being the lowest in the whole circuit) is represented by the point A. Between the Zn plate and the $ZnSO_4$ solution there is a sudden rise in potential of 0.52 volt, so that the potential of the $ZnSO_4$ solution next to the Zn plate is represented by the point B. Then follows a drop in potential in the cell until the Cu electrode is reached when a second sudden rise of 0.58 volt occurs, the potential of the $CuSO_4$ next to the copper plate, and that of the Cu electrode being represented by the points C and D respectively. From the copper plate, the potential falls off regularly owing to the external resistance R until the zinc plate is again reached at A'.

To complete the analogy, the figure should be considered as wound upon the surface of a cylinder so as to make A' coincide

with A. If both solutions are normal there is no appreciable difference of potential between the liquids themselves. The **electromotive force** E of a cell is the sum of the potential differences at the plates, *i. e.*, $AB + CD = AG$, while the value of the terminal potential difference E', is represented by FD, and depends upon the relative values of the external and internal resistances, R and r. In general this relation may be expressed thus:

$$E : E' :: R + r : R \tag{189}$$

whence

$$r = R \cdot \frac{E - E'}{E'}. \tag{190}$$

From the above equations it is evident that E' equals E only when R is infinitely great, and E' is zero for R equal to 0. In measuring the E. M. F. of a battery by means of a voltmeter it must be noted that the voltmeter measures only the difference of potential at its terminals, and that its resistance must be very large in comparison with r, if it is to give reliable values for E.

136. **Exercise 66. Terminal Potential Difference as a Function of the External Resistance.** Connect a voltmeter or galvanometer of high resistance to the terminals of a Daniell cell B, as shown in Fig. 100. The reading will give practically the E. M. F. of the cell or a deflection which is proportional to it. Close the circuit through R by means of the key K. The voltmeter will now read E'.

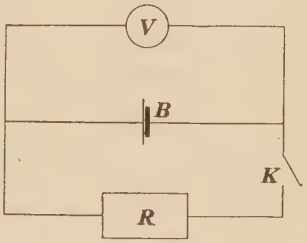

Fig. 100.

Vary the parallel resistance R, by steps as follows: 60, 40, 20, 10, 6, 4, 2, 1, and 0.5 ohms, and observe the corresponding values of E'. Plot E' and R as shown in Fig. 101. The curve will be an hyperbola, if r remains constant. Why? The **resistance r of the cell** may be **found directly from the curve,** for $r = R$ when $E' = E/2$. Calculate also the value of r from equation (190), and plot r as a function of I which equals E'/R.

If the resistance of the voltmeter be low, the external resistance should be calculated from the law of shunts.

FORM OF RECORD.

Exercise 66. To determine the terminal potential difference of a Daniell cell as a function of the external resistance.

Voltmeter No..... Date.....

	R	E'	r	I
Resistance box.....				
E (for R infinite).....				

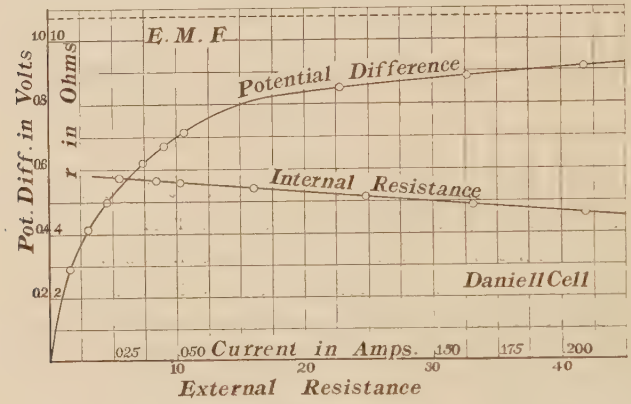

Fig. 101.

137. **Exercise 67. Electromotive Force and Internal Resistance by Voltmeter and Ammeter.** This method is a modification of the preceding. A milammeter is joined in series with the resistance box R in Fig. 100. The reading for E is made with the key open and then readings for E' and I are taken simultaneously, with K closed.

Since

$$\frac{E'}{R} = I \tag{191}$$

we have

$$r = \frac{E - E'}{I} \tag{192}$$

Vary the resistance of the ammeter circuit and take three different currents for each cell, but all of such a value that E' remains nearly equal to $E/2$. If R and the resistance of the ammeter be known, equation (190) may be used as a check formula, giving check values r', which should agree with those of r. In the case of dry cells the current is frequently so small that it can-

not be read accurately. In this case compute r from the values of E, E', and R. Since the polarization is quite rapid in some cells, it is better to read E immediately after the key K is opened, than at the beginning of each observation. The key is closed just long enough to take the readings.

FORM OF RECORD.

Exercise 67. To determine electromotive force and internal resistance of five different cells by voltmeter and ammeter.

Voltmeter No....	Name of cell	E	E'	I	$R+R_{am}$	r	r'
Ammeter No.....							
Resistance box...							

138. **Exercise 68. Electromotive Force and Internal Resistance by Condenser Method.** The foregoing method has two disadvantages: First, the **voltmeter reading** is only **approximately equal** to the E. M. F. of cells of **high internal resistance,** and second, the key K must **remain closed** until the readings can be taken, thus **permitting** of **considerable polarization.** In the **condenser method** the first objection is entirely overcome and the time needed for the observation is much shorter than in the previous method. The principle of this method is to obtain a deflection of a **ballistic galvanometer** proportional to the E. M. F. or to the difference of potential of the cell.

Two methods may be employed. (*a*) As shown in Fig. 102, a condenser of capacity C, and a ballistic galvanometer G are substituted for the voltmeter in Fig. 100. When the key K_1 is pressed down the condenser is charged with a quantity of electricity proportional to the difference of a potential at the terminals of the battery, and when the key swings back to the upper contact a deflection is obtained proportional to the charge, *i. e.*, proportional to the difference of potential of the battery terminals. In order to charge the condenser with the E. M. F. of the battery, keep key K_2 open and operate only key K_1. To

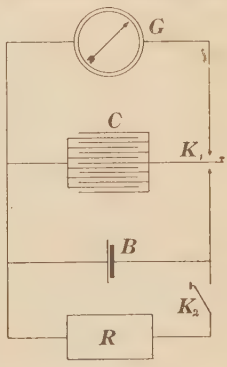

Fig. 102.

obtain deflections proportional to the difference of potential E' on closed circuit, close K_2, charge and discharge the condenser and immediately afterwards open K_2. Since the discharge is completed in a small fraction of a second it is not necessary to wait for the completion of the swing of the galvanometer coil before opening K_2.

A "pendulum apparatus" which automatically operates four keys, closing the circuit, charging and discharging the condenser and finally opening the circuit, is described in Carhart and Patterson's Electrical Measurements, pp. 107-109. This instrument is very useful for this exercise, not alone because it rapidly performs the four operations needed, but also because in successive observations always the same time interval elapses between the closing of the circuit and the charging of the condenser.

(*b*) The apparatus is arranged as in Fig. 103. The key K is a sliding contact which when passing from a to c makes for a short time connection between a and b and charges the condenser C through the galvanometer, the resulting deflection being proportional to the difference of potential at the terminals of the battery. To obtain a deflection proportional to E key K_1 should be open; to obtain a deflection with E', K_1 is closed, and K is turned from a through b to c. This closes the circuit as soon as the slider connects a and b and at the same time the condenser is charged. As the slider advances the galvanometer circuit is broken and shortly after also the current through R. Since very short time is needed to establish the current through R and charge the condenser fully, the momentary contact between a and b suffices. It is evident that there can be only slight polarization during this brief interval. The condenser is charged at the instant the circuit is closed.

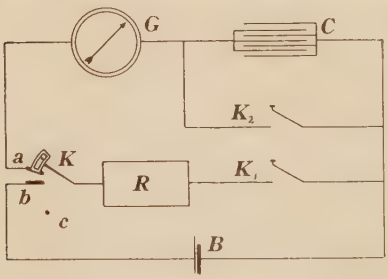

Fig. 103.

K_1 should be opened before the slider is pushed back to its original position. After each charge the condenser must be discharged

completely by the closing of key K_2, which should be kept closed for a short time to remove all absorbed charge if such be present.

In order to express the E. M. F. in volts, the condenser is also charged from a **standard cell** and discharged through the galvanometer. Let the deflection in this case be d_2, and let d be the deflection given by the cell under investigation when on open circuit, and d_1 when the cell is closed through the resistance R. The **deflections** are **proportional** to the **quantities** of **electricity passing** through the galvanometer, and these are themselves equal to the product of the difference of potential at the terminals of the condenser, times the constant capacity, C. Therefore if E_s be the E. M. F. of the standard cell, then

$$E = E_s \cdot \frac{d}{d_2}, \tag{193}$$

and

$$r = R \cdot \frac{d - d_1}{d_1}. \tag{194}$$

It should be observed that the **internal resistance** of batteries, and especially of dry batteries, appears **not to be a constant,** but to decrease with a decrease of R, that is with increasing current furnished by the cell. This is **probably due to polarization,** whose effect is neglected in the formulae given above [1] Use three different resistances for each cell, such that d_1 *is* about $d/2$.

FORM OF RECORD.

Exercise 68. To determine electromotive force and internal resistance of five cells by the condenser method.

Galvanometer..... Date.....

Condenser	Name of cell	d	d_1	R	E	r
Resistance						
Deflection, standard cell.....						
Temp. of standard cell.....						
E. M. F. of standard cell.....						

139. **Exercise 69. Internal Resistance by Method of Nernst and Haagn.** In this method the **internal resistance** of the cell is determined by means of an **alternating current,** while the cell itself furnishes **no current whatever.** In this way, if the

[1] Guthe, *Physical Review,* VII, p. 193, 1898.

period of the alternating current is sufficiently small, the disturbing effects of polarization are entirely avoided. The alternating current is furnished by the induction coil *I. C.* (Fig. 104), which charges the condenser *C* alternately to positive and negative potentials. These alternations are transmitted to a Wheatstone bridge, of which the condensers C_1 and C_2 form two arms, and the noninductive resistances R and r form the other two, where r is the resistance of the battery *B*. The resistance R is so adjusted as to produce **minimum sound** in the telephone receiver which replaces the galvanometer in the ordinary Wheatstone bridge.

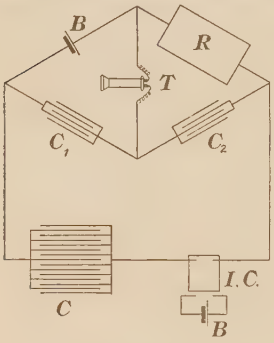

Fig. 104.

The applied difference of potential produces a current i, through the resistances R and r. Condenser C_1 will be charged with a quantity C_1ir, and condenser C_2 with a quantity C_2iR. For a minimum sound in the telephone these charges are equal or

$$C_1ir = C_2iR \qquad (195)$$

or

$$r = \frac{C_2}{C_1} R. \qquad (196)$$

In practice it is best to connect a slide-wire bridge in series with the battery and connect the telephone with the contact maker. We have then instead of r in one arm of the Wheatstone bridge, $r + R'$, where R' is the resistance of the part of the wire from the battery *B* to the contact maker. The resistance of the other portion of the slide wire forms of course part of R. The formula becomes in this case

$$r = \frac{C_2}{C_1} R - R'. \qquad (197)$$

FORM OF RECORD.

Exercise 69. To determine the internal resistance of five cells by the method of Nernst and Haagn.

Name of cell	C_1	C_2	R	R'	r

MEASUREMENTS OF CURRENT.

140. **Measurable Effects of a Current.** Since it is obviously impossible to preserve standards of current, it is also impossible to determine currents by direct comparison with a standard as in the case of resistance or electromotive force. It is necessary therefore to employ certain known effects of the current for purposes of measurement. The following are those chiefly used in laboratory practice. (*a*) **Electromagnetic effect** as applied in galvanometers, voltmeters and ammeters. These instruments have been in frequent use for the measurement of currents in the previous experiments. If any doubt as to their accuracy arises they should be calibrated by one of the methods which follow. (*b*) **Chemical effect,** as applied in the copper or silver coulometer. (*c*) The **production of a potential difference** at the terminals of a known resistance, as exemplified in Exercise 71.

141. **Law of Electrolysis.** According to Faraday's law[1] the quantity of a substance deposited by an electric current is proportional to the quantity of electricity passing through the electrolytic cell. A steady current of one ampere will deposit 4.025 grams of silver or 1.1838 grams of copper in one hour. The electrochemical equivalent z, of a substance is the ratio of the mass of the substance deposited to the quantity of electricity flowing through the coulometer. Since the quantity of electricity is the product of current and time it is easy to determine the **average current** flowing through an electrolytic cell, by dividing the number of grams of the substance deposited in time t by the weight which would have been deposited by unit current in the same time. Thus if w be the number of grams deposited, then

$$I = \frac{w}{z\,t}. \qquad (198)$$

Whenever it is desired to **calibrate** an **instrument** for measuring current by means of the coulometer, it is necessary to deter-

[1] *College Physics,* Article 283.

mine the **average reading** of the instrument **during** the **time** the current flows, and by comparing this reading with the average current, to determine the constant of the instrument or the correction to be applied in the case of direct reading instruments.

142. The Copper Coulometer. Of the various forms of coulometers the simplest is the copper coulometer (Fig. 105). Its manipulation demands considerable care. The coulometer consists of two copper electrodes in the form of plates or spiral wires, immersed in a solution of $CuSO_4$. This solution must be kept in a separate bottle and be used for this experiment only.[1] That part of the kathode on which the copper is to be deposited must be kept perfectly clean and *must not be touched with the fingers.* If not clean, the plate must be dipped into a strong solution of potassium cyanide then washed well with water and then dipped into strong alcohol and dried. All these operations should be performed as rapidly as possible since moist copper oxydizes easily in the air. In case the kathode is perfectly clean it may be used without further preparation.

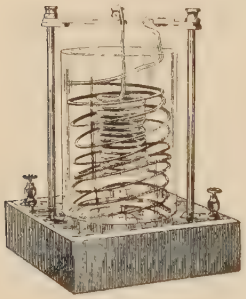

Fig. 105.

143. Exercise 70. Calibration of an Instrument by use of Copper Coulometer. First weigh the kathode to 0.1 milligram. Then set up the coulometers, two in series, in order to check the result. In the circuit place a battery of high, and if possible, of constant E. M. F., a large variable resistance, the instrument to be calibrated, and a key for opening and closing the circuit. The resistance should be adjusted beforehand so as to give the current that is to be sent through the coulometers. The current density may be as high as 1.5 amperes per square decimeter of the electrodes. Close the circuit for a definite time, say thirty minutes, and note the reading of the instrument every minute. After the circuit is opened wash the kathode in plenty of water, rinse in alcohol, dry and weigh to determine the mass of

[1] The following solution is recommended: $CuSO_4$, 15 grams; H_2SO_4, 5 grams; alcohol, 5 grams; water, 100 grams.

the deposit. Apply equation (198) for the calculation of the current.

FORM OF RECORD.

Exercise 70. To calibrate a.....by copper coulometer.

Date.....

Name and number of instrument.... Formula of instrument....

	Coulometer I	Coulometer II	t	Reading
Wt. of kathode before	.....	.....	.....	.....
Wt. of kathode after	.....	.....	.....	.....
Gain.....	.....			

Average gain..... Average reading of Inst.....
Average current..... Constant (or correction) of Inst..

144. Exercise 71. Calibration of Ammeter by Standard Cell. A current may be measured by comparing the difference of potential at the terminals of a known resistance r, through which it flows, with a known E. M. F. Let A (Fig. 106), be an ammeter to be calibrated; B a battery of constant E. M. F., and s an adjustable resistance for varying the current. R_1 and R_2 are two resistances, each very large in comparison with r. The standard cell is set in such a way as to oppose the difference of potential at the terminals of R_1. The resistances R_1 and R_2 are so adjusted that on closing K' after K, no current will flow through the galvanometer; then

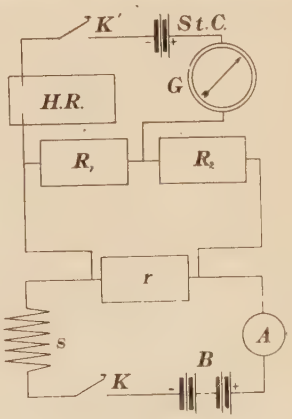

Fig. 106.

$$I r = i (R_1 + R_2) \qquad (199)$$

where I is the current flowing through r, and i that through R_1 and R_2. I is sensibly the same current as that through the ammeter.

Moreover

$$E_s = i R_1 \qquad (200)$$

therefore

$$I = \frac{E_s}{r} \cdot \frac{R_1 + R_2}{R_1} \qquad (201)$$

It is evident that under the conditions given the current must

be equal to or greater than E_s/r, in order to make a balance possible.[1] Calibrate an ammeter using at least five different currents.

FORM OF RECORD.

Exercise 71. To calibrate ammeter No.....

Date.....

Zero point of ammeter..... Resistance r.....

Standard cell No..... Temperature.... E. M. F.....

Ammeter reading		R_1	R_2	I computed	Correction
Observed	Corrected for zero pt.				

COMPARISON OF CAPACITIES.

145. **Exercise 72. Comparison by Direct Deflection.** The simplest method of determining the capacity of a condenser consists in comparing the deflections of a ballistic galvanometer (Fig. 74), caused by the discharge of the quantities of electricity stored in a standard condenser and in that under investigation, when each has been charged to the same difference of potential. From Exercise 50,

$$d_1 = Q_1/c = \frac{E C_1}{c} \tag{202}$$

$$d_2 = Q_2/c = \frac{E C_2}{c} \tag{203}$$

$$C_2 = C_1 \cdot \frac{d_2}{d_1} \tag{204}$$

One of the condensers C_1, is a standard condenser of known capacity.

[1] For a method for measuring small currents see Carhart and Patterson, *Electrical Measurements*, p. 172.

If the capacity of a condenser is so large as to give too large deflections, when discharged through the galvanometer, the experiment may be arranged as shown in Fig. 107. A battery of constant E. M. F. sends a current through two resistances R_1 and R_2, the sum of which must be kept constant. The condenser is connected over one of these resistances R_1, and charged with the difference of potential at its terminals equal to $i R_1$. If the deflection on discharging is too large the value of R_1 is to be reduced, R_2 being increased by the same amount. Then if C_1 and C_2 represent, respectively,, the standard and the unknown capacity

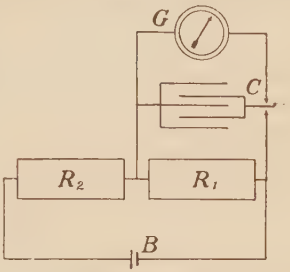

Fig. 107.

$$d_1 = \frac{i R'_1 C_1}{c} \text{ and } d_2 = \frac{i R''_1 C_2}{c} \qquad (205)$$

therefore

$$C_2 = \frac{d_2 R_1'}{d_1 R_1''} \cdot C_1 \qquad (206)$$

Instead of two resistance boxes the wire of a slide wire bridge may be used.

Not infrequently condensers are found whose capacity depends very largely upon the time of charge. Such condensers are termed **absorbing condensers.** The quantity of electricity stored up in such condensers may be considered as consisting of two parts (*a*) the **free charge,** (*b*) the **absorbed charge.** In discharging through a galvanometer the free charge will pass first and then the absorbed charge will follow. Part of the latter will affect the throw of the galvanometer and the "effective" capacity will therefore depend to a certain degree upon the period of the galvanometer, being the larger the larger the period. To obtain results independent of the period the discharging circuit must be opened a few hundredths of a second after contact with the galvanometer is made.[1] Thus we measure only the free charge which in absorb-

[1] Zeleny, *Physical Review,* Vol. 22, p. 65, 1906.

ing condensers is frequently less than one half of the quantity of electricity observed by applying the above method after long continued charging. To study the effect of absorption the student should take different intervals for charging and note carefully the resulting effect upon the observed capacity.

FORM OF RECORD.

Exercise 72. To measure the capacities of four condensers.

Date.....

Galvanometer.....	Name of condenser	Time of charge	R_1	d	C
Resistance boxes..					
Cell..					

146. Exercise 73. Method of Mixtures. The apparatus is arranged as shown in Fig. 108. In the middle of the figure is

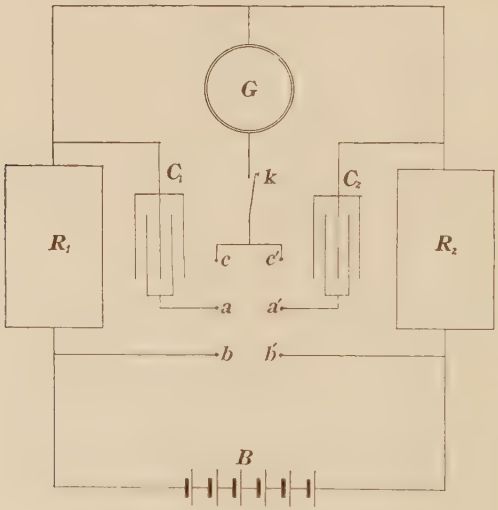

Fig. 108.

shown a double throw switch, by means of which the points a and a' may be connected either to b and b' or to c and c'. By means of the first connections the condensers C_1 and C_2 are charged with quantities of electricity equal to $R_1 i C_1$ and $R_2 i C_2$ re-

spectively. On reversing the switch these quantities mix and there will, in general, be a deflection of the galvanometer resulting from the difference of these two charges on pressing the key k. Adjust the resistances R_1 and R_2 until there is no deflection of the galvanometer; then

$$R_1 i C_1 = R_2 i C_2 \qquad (207)$$

and

$$C_2 = C_1 \cdot \frac{R_1}{R_2} \qquad (208)$$

The **battery** should have a **relatively high E. M. F.**; six to ten Leclanché elements may be used. The disturbing influence of absorption may be studied as in the preceding experiment.

FORM OF RECORD.

Exercise 73. To compare two condensers by the method of mixtures.

Date.....

Galvanometer.....	Name of condenser	Time	R_1	R_2	C
Resistance boxes.....	.....	.....	.....	.....	.....
Standard condenser.....	.....	.....	.....	.....	.....

MEASUREMENT OF INDUCTANCE.

147. **Exercise 74. Selfinductance of a Coil Compared with a Standard.** Arrange the apparatus as in Fig. 109, in which the two selfinductances form each one arm of a Wheatstone bridge. Let their resistance and selfinductance be R_1 and L_1, R_2, and L_2 respectively. The remaining two arms are formed by the **noninductive resistances** R_3 and R_4. First adjust R_3 and R_4 for *constant current* until the galvanometer shows no deflection on closing k, that is,

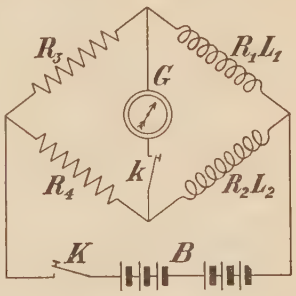

Fig. 109.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \qquad (209)$$

Then, **keeping k closed**, open K. There will in general be a de-

flection of the galvanometer due to the difference of the E. M. F. produced by the selfinductances in the branches L_1 and L_2. Vary the selfinductance of the standard, until upon opening K no deflection is obtained. Then the points connected to the galvanometer will be at the same potential as well during the steady as during the variable state of the current.

Let the difference of potential over R_1 or R_2 for a constant current be E; then the current through R_1 is $i_1 = E/R_1$, and through R_2, $i_2 = E/R_2$. Now the value of the E. M. F. due to selfinductance is[1]

$$e = -L \cdot \frac{di}{dt}. \tag{210}$$

In the case under consideration, the value of e on opening K at any moment, is

$$e_1 = -L \cdot \frac{di_1}{dt} = -\frac{L_1 dE}{R_1 dt} \tag{211}$$

and

$$e_2 = -L_2 \cdot \frac{di_2}{dt} = -\frac{L_2 dE}{R_2 dt}. \tag{212}$$

But after the adjustment of the standard, e_1 equals e_2 and dE/dt is the same for both e_1 and e_2, and hence the condition for no deflection during the variable state of the current is given by the equation

$$\frac{L_1}{R_1} = \frac{L_2}{R_2} \text{ or } L_1 = L_2 \frac{R_1}{R_2} = L_2 \frac{R_3}{R_4}. \tag{213}$$

In practice care must be taken to place the two selfinductances in such a position that they will not influence each other. To obtain accurate results the battery must possess a high voltage. The best results are obtained by using an alternating current and substituting for the galvanometer a telephone receiver as in Exercise 61, or by rectifying the current before sending it through the galvanometer. In an instrument designed by Ayrton and Perry, called the **secohmmeter**,[2] this rectification of the current is effected by means of a double commutator.

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[1] *College Physics*, Article 333.

[2] Carhart and Patterson, *Electrical Measurements*, p. 110.

FORM OF RECORD.

Exercise 74. To measure the selfinductance of.....

Date.....

	R_3	R_4	Reading of standard	L
Apparatus	.....	.....	.....	.....
Voltage of battery.....	.....	.....	.....	.....

148. **Exercise 75. Mutual Inductance of two Coils.** According to definition the **coefficient of mutual induction** or, the **mutual inductance** of two coils is given by the equation[1]

$$e = -M \frac{di}{dt}. \tag{214}$$

where e is the counter E. M. F. in the secondary due to the mutual inductance M, and i, the current in the primary. Let the induced E. M. F. produce a current through a ballistic galvanometer whose resistance is g; the current i', so produced, will at any moment be $\frac{e}{r+g}$, where $r+g$ is the total resistance of the secondary circuit. The total quantity of electricity passing through the galvanometer is

$$\int i' dt = \int \frac{e}{r+g} dt = -\frac{M}{r+g} \int_0^I di, \tag{215}$$

where the integral is to be taken between o and I, the final value of the steady current in the **primary circuit.** Therefore the total quantity in the secondary circuit when the primary circuit is closed, becomes, neglecting its direction

$$Q = \frac{M I}{r+g} \tag{216}$$

If the current through the primary be suddenly reversed the value of Q is doubled. From the formula for the ballistic galvanometer we have $Q = cd$, where c is the constant of the galvanometer. Therefore

$$d = \frac{M I}{c(r+g)} \text{ and } M = \frac{c(r+g)d}{I} \tag{217}$$

[1] *College Physics, Article* 332.

This formula is based upon the assumption that no counter electromotive force is generated in the galvanometer. But this assumption is *not correct,* especially in the case of instruments of the d'Arsonval type. In this case a counter electromotive force is set up, due to the cutting of lines of magnetic induction by the coil swinging in the field of the permanent magnet. Since the total number of lines cut, or the change of flux through the coil is nearly proportional to the actual deflection d, the resulting quantity of electricity[1] is nearly equal to $\frac{k\,d}{c(r+g)}$, and tending to flow in the opposite direction, would, if acting alone on the circuit, produce a deflection $-d_2$, in the opposite direction from d, while if there were no inductive effect in the coil, the deflection would be simply

$$d_1 = \frac{M\,I}{c(r+g)} \qquad (218)$$

We obtain, therefore, for the actual deflection

$$d = d_1 - d_2 = \frac{I}{c(r+g)}\ (M\,I - k\,d) = \frac{M\,I}{c(r+g+\frac{k}{c})} \qquad (219)$$

In order to obtain accurate results we must therefore add the quantity k/c to the actual resistance $r+g$; in other words, we must here use the **apparent resistance** of the galvanometer,

$$g' = g + \frac{k}{c}, \qquad (220)$$

instead of its **ohmic resistance** g.

The expression for M thus becomes

$$M = \frac{c(r+g+\frac{k}{c})\,d}{I} = \frac{c(r+g')\,d}{I}. \qquad (221)$$

The apparent resistance g', of the galvanometer may be determined in the following manner. As described above, produce a deflection of the galvanometer, and note the deflection. Let the total resistance of the secondary circuit be $r+g$, where r denotes

[1] *College Physics,* Article 330.

the resistance outside the galvanometer. Next introduce in series a resistance r', such that with the same primary current the deflection is now reduced to one half its former value. Then

$$\frac{c(r+g')d}{I} = \frac{c(r+r'+g')}{I}\frac{d}{2}$$

and

$$g' = r' - r \qquad (222)$$

The deflection is proportional to the current in the primary and inversely proportional to the resistance of the secondary circuit. To show these relations clearly make the following experiments: The arrangement is shown in Fig. 110. Let P represent the primary and S the secondary coil. The primary is in series with a battery B, a variable resistance R and an ammeter A. To the secondary coil are joined a resistance r and a ballistic galvanometer.

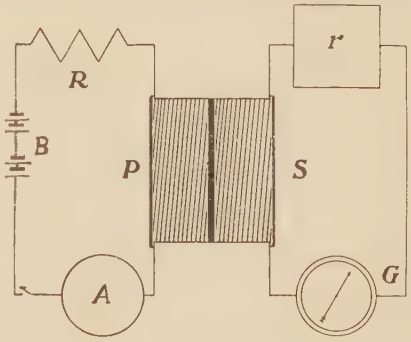

Fig. 110.

(1) Vary I keeping r constant. Plot values of d and I.

(2) Vary r keeping I constant. Plot d and $1/(r+g')$; both curves should be straight lines.

FORM OF RECORD.

Exercise 75. To determine the mutual inductance of.....

Date.....

Ammeter..... Ballistic galvanometer.....

Determination of constant:	(1) r constant			(2) I constant		
Capacity	I	d	M	r	d	M
Electromotive force	.....	.....	.....	.....	.....	.....
Deflection	.....	.....	.....	.....	.....	.....
Constant	.....	.....	.....	.....	.....	.....

Apparent resistance:

r..... d_1.....

r'..... d_2.....

g'.....

CHAPTER IX.

MAGNETIC MEASUREMENTS.

MAGNETIC FIELDS.

149. Magnetic Fields. In most of the foregoing experiments the action between a magnetic field produced by a current and another magnetic field was used to determine electrical quantities. In the following experiments magnetic fields and their effect upon the magnetic properties of iron and steel will be studied. A magnetic field is perfectly determined if at every point the intensity H of the field and the magnetic induction $B = \mu H$ of the field be known. The **intensity** may be compared to a stress in an elastic body, the **induction** to a **strain**[1]. In the case of air, since μ equals unity, these two quantities are numerically equal, but it should be kept in mind that they are different physical quantities.

The intensity of a magnetic field at a given point may be measured by the force per unit pole strength acting on a pole placed at this point. The unit of intensity of magnetic field is the dyne per unit pole and is called the **gauss.** If the magnetic field be produced by an electric current it is best to calculate the intensity at any point from the current. In the case of a circular current of radius r it is[2]

$$H = \frac{2\pi}{10\,r}\ I \text{ gauss} \qquad (223)$$

In the case of a very long solenoid it is[3]

$$H = \frac{4\pi n'}{10}\ I \text{ gauss} \qquad (224)$$

where n' is the number of turns of wire per unit length of the solenoid. In both equations I is expressed in amperes.

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[1] *College Physics,* Article 242.
[2] *College Physics,* Article 257.
[3] *College Physics,* Article 319.

The magnetic induction B at any point is measured by the number of lines of induction per unit crosssection and its unit is therefore the **line per square centimeter.** The magnetic flux Φ through a given area is simply the number of lines of induction through this area A, or[1]

$$\Phi = B\,A \qquad (225)$$

150. **Exercise 76. Determination of H. (First Method).** The lines of magnetic induction due to the earth's field run from South to North, although deviating in some places by an appreciable angle from the geographical North and South line. This angle is called the **angle of declination.** The lines of induction are also inclined towards the horizontal plane, making in Ann Arbor an angle of $72°$ with the horizon. This is the angle of magnetic inclination or the **magnetic dip.** It should be noted that neither the magnetic declination nor the dip is constant. They not only vary from place to place over the earth's surface, but they also vary slightly from year to year and from day to day in the same place. The **magnetic field** of the earth may be conceived as the **resultant of two components,** one horizontal H, and one vertical V. Then Θ, the angle of dip, is given by the equation

$$\tan \Theta = \frac{V}{H}. \qquad (226)$$

It is of great interest to determine the value of the horizontal component of the earth's magnetism in terms of the fundamental units of length, mass and time, since all the practical magnetic and electrical units are based on these[2].

A relation between the magnetic moment of a magnet and the

[1] *College Physics,* Article 324.
[2] *College Physics,* Articles 258 and 404.

strength of the magnetic field in which it is situated may be derived in either one of two ways:

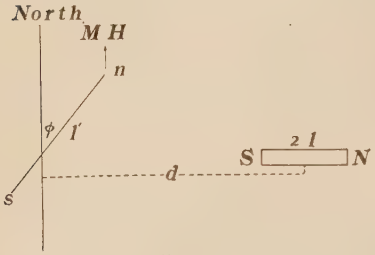

Fig. 111.

(1) *Method of deflections.* Let a magnet *NS*, (Fig. 111) be placed with its axis on the magnetic East-West line[1] and on this line in the same horizontal plane, at a measured distance from it place a very small magnetic needle *ns*, suspended by a fine silk thread or quartz fiber.

Let m and l be the pole strength and half-length of magnet NS. Let m' and l' represent the same for magnet $n\,s$.
Also let
d be the distance between the centers of the magnets,
H the horizontal component of the earth's magnetic field,
M the magnetic moment of the magnet $NS = 2ml$,
M' the magnetic moment of the magnet $n\,s = 2m'l'$.

The magnet *ns* will be deflected from its normal position by the angle ϕ such that the turning moments due to the earth's field and that due to the influence of the magnet NS are equal[2]. The force due to the earth's field on one of the poles of *ns* is $m'H$, and the lever arm on which it acts is $l'\sin\phi$; so the turning moment on the whole magnet is

$$2m'l'H\sin\phi = M'H\sin\phi\,. \qquad (227)$$

The force on the north-seeking pole of *ns* due to the south-seeking pole of *NS* is, according to Coulomb's law of the inverse squares $\frac{1}{\mu}\frac{mm'}{(d-l)^2}$, l' being considered negligible in comparison

[1] To find the magnetic East-West line, suspend in the center of a plane coil of wire a magnetic needle. Turn the coil until on sending a current through it the needle is not deflected. The plane of the coil is then in the magnetic East-West line.

[2] The angle ϕ is determined by mirror and scale. Suppose the distance of the mirror from the scale to be D and the deflection d, then $\tan 2\phi = d/D$.

with d. The force of the north-seeking pole of NS is in the opposite direction and equal to $\frac{1}{\mu}\frac{m\,m'}{(d+l)^2}$, so that the whole force on the north-seeking pole of the needle is:

$$F_1 = \frac{m\,m'}{\mu}\cdot\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right] \qquad (228)$$

and if l be small in comparison with d, and since μ is unity,

$$F_1 = \frac{4\,m\,m'\,l}{d^3}. \qquad (229)$$

The turning moment due to this force is $F_1 l' \cos\phi$, and since the turning moments on the two poles of ns are equal and in the same direction the total turning moment exerted on ns by the magnet NS is $2\,F_1 l' \cos\phi$ or

$$\frac{8\,m\,m'\,l\,l'}{d^3}\cos\phi = 2\cdot\frac{M\,M'}{d^3}\cos\phi. \qquad (230)$$

But the turning moments produced by the forces exerted by the earth and the magnet NS must be equal since the needle is in equilibrium. So the expressions for these turning moments may be set equal to each other, or

$$M'\,H\sin\phi = 2\,\frac{M\,M'}{d^3}\cos\phi \qquad (231)$$

whence

$$\frac{M}{H} = \frac{d^3}{2}\cdot\tan\phi = A. \qquad (232)$$

(2) *Method of oscillations.* The law of vibration of a magnetic needle in a magnetic field is the same as that of the physical pendulum. If we suspend the deflecting magnet NS, so as to swing freely, its period is

$$T = 2\pi\sqrt{\frac{K}{M\,H}}, \qquad (233)$$

where T is the period of a complete vibration and K is the moment of inertia of the magnet. MH is the torque when the

magnet is at right angles to the force. In order to find K the same method is applied as in Exercise 26, by putting a brass ring of known moment of inertia K' on the magnet so that the axis of rotation and the axis of the ring coincide. Let the period of vibration of the new system be T', then

$$T' = 2\pi\sqrt{\frac{K+K'}{MH}}, \tag{234}$$

and from the equations (233) and (234)

$$MH = \frac{4\pi^2 K'}{T'^2 - T^2} = B. \tag{235}$$

Equations (232) and (235) furnish two expressions involving M and H.

$$\frac{M}{H} = A \tag{236}$$

and

$$MH = B,$$

from which

$$H = \sqrt{\frac{B}{A}}. \tag{237}$$

or on substituting the values of A and B,

$$H = \frac{2\pi}{d}\sqrt{\frac{2K'}{d(T'^2 - T^2)\tan\phi}}. \tag{238}$$

It is obvious that by this method the value of H is determined in the fundamental units of mass, length and time.

The instrument used for this experiment is called a magnetometer. It consists of a closed case furnished with windows to enable the observer to measure the deflection of the needle by means of the mirror which it carries. Place the deflecting magnet at a certain distance from the small needle as described above and observe the deflection. Turn the magnet end for end and again observe the deflection, which will now be in the opposite direction. Repeat these two operations with the magnet at the

same distance on the opposite side of the needle and take the mean of the four observations as the deflection ϕ. In case the length of the deflecting magnet is not negligible in comparison with the distance d, we may take it into account. Kohlrausch has shown that for a bar magnet the distance between the poles is very nearly 5/6 of the length of the magnet and this value should be used for $2l$. The formula for H becomes

$$H = \frac{2\pi}{d^3 - l^2}\sqrt{\frac{2K'd}{(T'^2 - T^2)\tan\phi}} \qquad (239)$$

In the vibration method the time of vibration may be determined as in Exercise 13, or by the use of an ordinary stop watch if less accuracy is required. The stop watch should however be compared with a standard clock. How may the magnetic moment of the deflecting magnet be determined from the foregoing formulae?

FORM OF RECORD.

Exercise 76. Determination of H.

(1) Date.....

Determination of $\frac{M}{H}$	Position of magnet.	Deflection	tan 2 ϕ	tan ϕ
Distance d.....	a	.....	.....	.....
Length of magnet.....	Reversed	.....	.....	.....
	b	.....	.....	.....
	Reversed	.....	.....	.....

Computation of $\frac{M}{H}$

(2) Determination of MH:

(a) Determination of T.
(b) Determination of T'.
(c) Data for moment of inertia of ring.
Mass of ring.... Outer diameter of ring.... Inner....
Moment of inertia of ring.....
Computation of MH.

3) Computation of H.

$H =$ .....

151. Exercise 77. Determination of H. (Second Method). In formula (233),

$$T = 2\pi\sqrt{\frac{K}{MH}}$$

the torsional moment of the suspending wire was neglected. The complete formula for a given magnet is

$$T = 2\pi\sqrt{\frac{K}{MH + \mathscr{T}_1}}$$

or

$$\frac{1}{T^2} = Hc + c' \tag{240}$$

where $\mathscr{T}_1$ is the moment of the torsional couple of the suspending wire, and c and c' are constants. These relations are illustrated in the following experiment. Let a short magnetic needle be suspended at the center of a solenoid whose length is at least twenty times its diameter, and whose axis is parallel to the magnetic meridian. Observe the period of vibration of the needle, first when swinging in the earth's field H, and then when a magnetic field H_1, due to a known current I through the soleniod, is superposed upon the field of the earth. Let the period of vibration of the system in the first case be T and in the second T_1. Then

$$\frac{1}{T^2} = Hc + c' \tag{241}$$

$$\frac{1}{T_1^2} = c(H + H_1) + c' \tag{242}$$

Vary the current and observe T_1, T_2, T_3, T_4, etc.; also the periods of vibration T'_1, T'_2, T'_3, etc., when the current through the solenoid is reversed. Then plot $1/T^2$ as ordinates and the magnetic field strength produced by the currents as abscissæ, the latter positive or negative according to its direction[1]. The curve consists

[1] If the current be increased in the negative sense beyond a certain value the magnet turns through 180°. Why?

of two straight lines meeting at the point O', Fig. 112. The time corresponding to the ordinate $O\,B$ is then the period of vibration corresponding to the rigidity of the suspension alone. If O' be considered as the origin and the strength of field be plotted on $O'x$ it is evident that $O'B$ is the value of the horizontal component of the earth's field, and that the strength H' of any field may be readily determined by allowing the magnet under experiment to vibrate in this field and determining its period of vibration. Then

$$H' = \frac{1}{T^2 c} - \frac{c'}{c}, \qquad (243)$$

where c and c' are to be substituted from the foregoing observations.

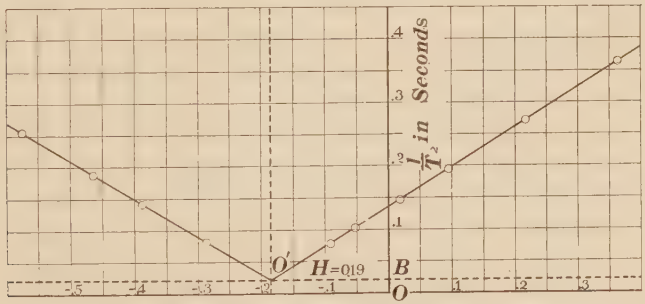

Fig. 112.

In practice a storage cell of constant E. M. F. is used to furnish the current through the solenoid. A resistance is joined in series to allow the current to be varied within wide limits. The periods of vibration may be determined by means of a stop watch. From equation (224) the strength of field at the center of the solenoid is

$$H_1 = \frac{4\pi N I}{10 L},$$

where N is the number of turns of wire and L the length of the solenoid. The current I is computed from the electromotive force of the cell and the total resistance.

FORM OF RECORD.

Exercise 77. Determination of H. (Second Method).

Dimensions of solenoid:	R	I	T	$\frac{I}{T^2}$
Length	.....	.....	.....	.....
$n =$	.....	.....	.....	.....
Determined from curve:	.....	.....	.....	.....
$c =$	.....	.....	.....	.....
$c' =$	.....	.....	.....	.....
$H =$	.....	.....	.....	.....

MAGNETIC PROPERTIES OF IRON AND STEEL.

152. **Magnetizing Field and Permeability.** When a bar of unmagnetized iron or steel is introduced into a magnetic field the distribution of lines of magnetic induction is greatly altered. The number of lines per square centimeter at any point in the bar is considerably larger than it was at the same point before the introduction of the iron. In other words B which is called the **induction** is much increased. At the same time the intensity of the field inside the iron is, in general, much smaller than it was before at that place. This intensity which is called the **magnetizing intensity** may be considered as being the result of the superposition of the **original field** and the **demagnetizing effect** of the ends of the iron bar. This effect is in the opposite direction to the original field and thus the intensity of the field is reduced. The demagnetizing effect of the ends may be neglected only in the case of a long bar. An excellent method of avoiding this disturbing influence of a magnetized piece of iron consists in using a ring instead of a bar and in producing the field by means of a solenoid wound around this ring. Since there are no ends in this case the magnetizing field is exactly equal to the field calculated from the equation (224)

$$H = \frac{4\pi n'}{10} I = \frac{4\pi N}{10 L} I$$

where N is the total number of turns of wire and L the length of the ring shaped solenoid which is equal to 2π times the average radius of the ring.

The **magnetic permeability** μ **of iron** is the ratio between the

[1] For a more complete treatment of the properties of ferromagnetic substances and a description of the various methods of measurement, see J. A. Ewing, *Magnetic Induction in Iron and other Metals.*

induction produced and the magnetizing intensity of the field, or

$$\mu = \frac{B}{H} \qquad (244)$$

As shown in exercise 79 this ratio depends greatly upon the previous history of the piece under experiment. In order that the permeability of a substance shall give definite information concerning its magnetic quality, it has been agreed to express the permeability of iron and similar substances as the above ratio when the substance is originally unmagnetized and then subjected to an increasing magnetizing intensity, (Exercise 78). Even under these conditions the permeability of a given sample of iron is not a constant but varies with the magnetizing intensity.

Under the influence of variable or alternating magnetic fields iron and steel show what is termed **magnetic hysteresis,** and the hysteresis curve (Exercise 79) should be determined for all material which is intended to be used in alternating current apparatus, for example in alternators or transformers.

All substances which have a large permeability approaching in magnitude that of iron are called **ferromagnetic substances.**

153. **Exercise 78. Commutation Curve for Iron and Steel.** It is of interest to determine the dependence of B upon the intensity of the magnetizing field produced by the current alone, when the intensity is slowly increased from 0 to 30 or 40 gauss. It will be seen from Fig. 113, that B increases slowly at first, then very rapidly, then slowly again, until it finally increases at the same rate as H; that is, the presence of the iron does not introduce any additional lines of induction. The iron is then said to be saturated.

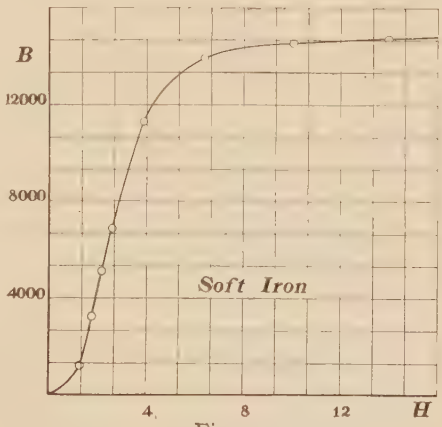

Fig. 113.

The method here described is known as the **ballistic** or **ring method** and has the **advantage,** that the magnetic circuit contains **no air gaps** which offer large magnetic resistance and therefore exert strong demagnetizing effects. (Article 152). The metal is given the form of a ring of uniform cross section, Fig. 114. It is best to have the cross section of the ring approach the form of a rectangle and the diameter large as compared with the thickness.

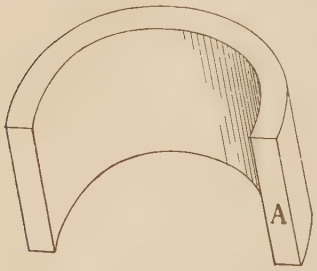

Fig. 114.

The outer and inner diameters of the ring must be measured with care, and the volume of the ring obtained from its loss of weight in water. The cross section A is readily found from the volume by dividing by the average circumference. The ring is then covered with insulating tape upon which is wound a layer of wire covering the entire ring and forming the primary coil. The number of turns N_1, in the primary coil must be carefully determined. Over this is wound the secondary coil, five to twenty turns, N_2. The apparatus should be arranged as shown in Fig. 115. In the primary circuit are joined in series a storage battery, an ammeter, a rheostat R, the primary coil of the ring and a commutator for reversing the current through the coil. The secondary coil is connected to a **ballistic galvanometer** whose **constant** and **apparent resistance** are known (see exercise 75). It is most convenient to make the total galvanometer resistance the critical resistance (see page 142).

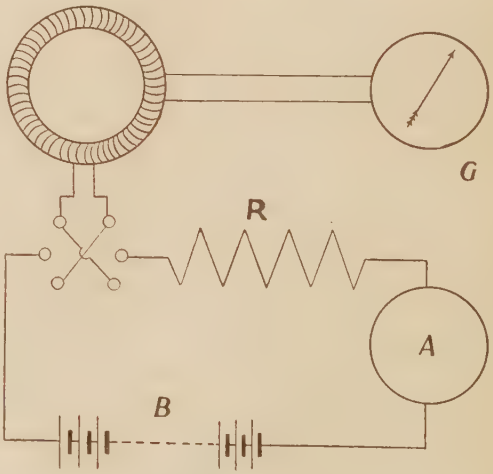

Fig. 115.

The constant may also be determined by means of a known mutual inductance M; in this case $c = \frac{M I}{d r_2}$, where d is the deflection produced in the secondary by closing the current I in the primary. In this case, however, the total resistance of the secondary must remain the same, either by keeping the secondary of the induction coil in the circuit or by substituting an equal resistance for it. What formula must then be used instead of (249)? What is the **numerical factor,** if M be expressed in **millihenrys** (10^6 c. g. s. units) and I in **amperes** (10^{-1} c. g. s. units)?

If we vary the number of lines of magnetic induction in the solenoid, the electromotive force induced in each turn of the secondary is at any moment equal to the rate of change of the flux threading through the circuit, or

$$e = -\frac{d\Phi}{dt}. \tag{245}$$

If there are N_2 turns of wire in the secondary coil then

$$e = -N_2 \cdot \frac{d\Phi}{dt}. \tag{246}$$

The current i, passing through the galvanometer is then

$$i = -\frac{N_2}{r_2}\frac{d\Phi}{dt} \tag{247}$$

where r_2 is the total resistance of the secondary circuit. The total quantity of electricity Q, passing through the ballistic galvanometer is $\int i dt$, corresponding to a change of Φ lines of induction, or

$$Q = \frac{N_2 \Phi}{r_2} = c\,d \tag{248}$$

where c is the constant of the galvanometer and d the throw. Express c in c. g. s. units. Since the constant is usually given in micro-coulombs per scale unit, one micro-coulomb being 10^{-7}

c. g. s. unit, and the resistance in ohms, one ohm being equal to 10^9 c. g. s. units, the formula becomes

$$\Phi = 100 \cdot \frac{c\, d\, r_2}{N_2} . \qquad (249)$$

Since B denotes the number of lines of induction per unit area the increase of B corresponding to a deflection d of the galvanometer when the current is varied in the primary, is given by

$$\Delta B = \frac{\Phi}{A} = 100 \cdot \frac{c\, d\, r_2}{N_2 A} . \qquad (250)$$

To obtain the **commutation** curve it is important to start with an unmagnetized ring. It is best to demagnetize the ring by sending first a rather large current, say three amperes, through the primary, increasing the resistance R, and then reversing the direction of the current. This must be done several times, taking care to decrease the current each time before the reversal, until the current has become very small. On breaking the circuit the ring will contain no residual magnetism. A more convenient way is to connect the primary to the terminals of a small alternator driven by a belt. Let the alternator attain its full speed and then throw off the belt. As the speed decreases the current decreases, and the iron is as before subjected to cycles of constantly decreasing magnetic intensity.

Arrange the apparatus, as shown in Fig. 115. Adjust the resistance so as to give a small current through the primary coil. Reverse the current and observe the first ballistic throw of the needle d'_1 reverse again and observe d''_1; the mean, d_1, corresponds to a reversal of the current I, read from the ammeter. This is repeated, increasing the current step by step, until the deflections increase very slowly with increasing current. Compute H from equation (224), and B from (250), remembering that but **half the average deflection must be taken,** since the **current is changed** each time by $2I$ instead of by I.

Plot B and H; the curve will resemble that shown in Fig. 113. From the curve it appears at once that μ is not a constant, but varies greatly with the degree of magnetization.

FORM OF RECORD.

Exercise 78. Commutation curve for iron.

Date.....
Galvanometer..... Ammeter No.....
Constant of galvanometer Resistance r_2.....
Ring..... Outer diam..... Inner..... Average.....

Volume of ring.....	I	d'	d''	d	$d/2$	H	B	μ
Crosssection.....								
Turns in primary.....								
Turns in secondary.....								

154. **Exercise 79. Hysteresis Curve for Iron or Steel.** If instead of reversing the current in the previous exercise we should begin with a large current, decrease I by small steps, observe the corresponding throws of the galvanometer and plot B with respect to H, we obtain an entirely new curve owing to the effects of **residual magnetism** in the iron or steel. The experiment thus allows us to study the lagging of the induced magnetism behind the magnetizing field intensity. Thus if, in the beginning, the iron were well magnetized, the starting point on the curve would be the highest point to the right, (Fig. 116), and on reducing the current to zero, there would still remain in the iron the lines of induction represented by the positive ordinate. The iron is still magnetized, but on reversing the current the value of B decreases very rapidly, becomes zero for a small negative cur-

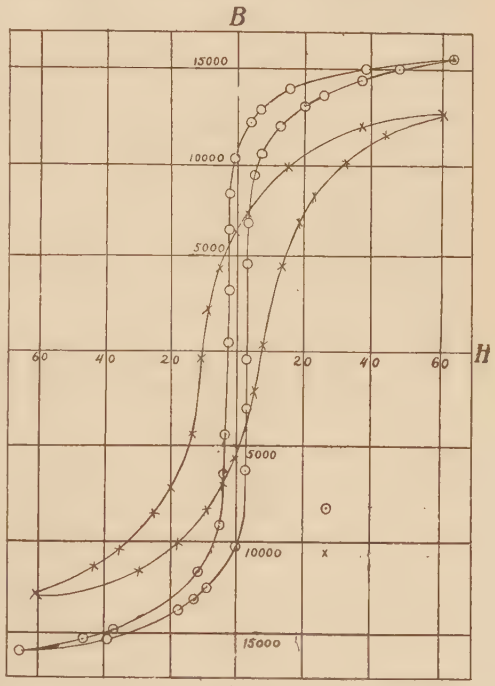

Fig. 116.

rent, and finally reaches a value symmetrical to that at the starting point, when the current in the negative direction reaches the same value it had at the beginning in the positive direction.

On decreasing the current again to zero and increasing it in the positive direction to the original value, the curve will have a form resembling closely the first branch. The closed curve is called a **hysteresis curve.** The value of B for zero current is termed the **remanence** and the value of H for zero B is called the **coercive force** of the iron.

The arrangement of the apparatus is similar to that used in the preceding exercise. The commutator in the primary is connected as shown in Fig. 117. Start the experiment with the commutator in the position indicated by the dotted lines and with switch K closed. Adjust rheostat R, until the largest current to be used in the experiment is reached. The iron has then attained its **maximum magnetization,** indicated by the **highest point** on the hysteresis curve.

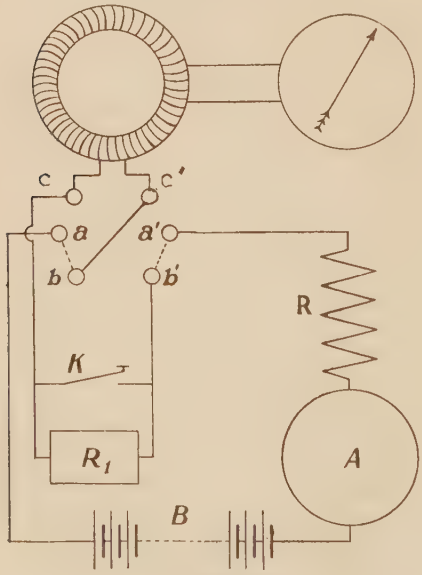

Fig. 117.

If now K be opened, the current is suddenly decreased, owing to the addition of resistance in rheostat R_1. Adjust R_1 to different values, beginning with small resistance and increasing them for every following observation. We obtain every time on opening K a deflection corresponding to the total change in the induction, due to the decrease of current to a value I, determined by the resistance in R_1. Finally the current is broken by making R_1 infinite. In this way we obtain points on the hysteresis curve corresponding to a decrease of the magnetizing field from a maximum value to zero.

To obtain the points corresponding to a reversal of H, the commutator is connected so as to join a to c, and a' to c'. R remains adjusted for the largest current and the commutator is then suddenly reversed. In this way a magnetic field is produced in the iron in the opposite sense, the strength of the field depending upon the resistance in R_1. This resistance should now be reduced from very high values to smaller and smaller ones and finally to short circuit. The deflections d will now be in the opposite direction to that before, unless the galvanometer or the battery connection be reversed. It is evident that by this method we obtain points for but one-half of the hysteresis curve. Complete the curve by drawing the other half. The origin of the axis of induction B, is found by taking as the ordinate for maximum induction the $\triangle B$ corresponding to one-half of the maximum deflection. Use equation (250).

FORM OF RECORD.

Exercise 79. Hysteresis curve for iron or steel.

Date.....

Record apparatus as in preceding exercise.	I	H	d	ΔB	B

CHAPTER X.

OPTICAL MEASUREMENTS.

CURVATURE.

155. Curvature of Optical Surfaces. It is shown in treatises on optics that the effect of a **mirror** or of a **lens** of any form, consists in impressing upon the wave-front of the luminous disturbance a **curvature** directly related to the **curvature of the mirror or lens** in question. By definition the **curvature** at any point in a curve is the **reciprocal** of the **radius** of the **osculating circle** at that point. Since the effects produced by mirrors or lenses are to be predicted from a knowledge of their constants, it becomes a matter of importance to measure the curvature of an optical surface, in other words to determine its **radius of curvature.**

The **surface most commonly employed** in optical construction is that of the **sphere** since only the largest mirrors or lenses possess surfaces differing noticeably therefrom. **Concave** or **convex mirrors** may therefore be regarded as **parts of spherical shells,** with the inner or outer surface polished as the case may be. The **radius of curvature** of such a mirror is obviously the **radius of the sphere,** of which the mirror forms a part. In lenses both surfaces are to be regarded as parts of spheres of definite radii. In the case of a plane surface the radius is of course infinite.

156. Exercise 80. Radius of Curvature of a Lens by the Spherometer. The experiment consists in determining the radius of curvature from a careful measurement of the amount by which the lens surface departs from a plane, *i. e.,* by measuring the **sagitta**[1]. If we place a spherometer upon a lens with the three feet resting upon the surface of the lens, we may imagine a plane passed through the lens, cutting from it a seg-

[1] Preston, *Theory of Light,* p. 80.

ment whose base is a circle passing through the three feet of the instrument. At right angles to the base of this segment stands the micrometer screw of the spherometer, and by taking readings, first upon a plane surface and then upon the lens, the sagitta of the curve, that is the distance the central foot of the instrument is above or below the plane containing the other three feet, may be accurately determined.

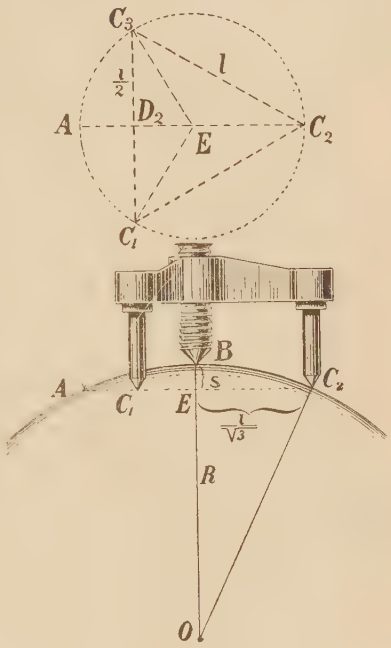

Fig. 118.

Thus in Figure 118 is shown in perspective the lens, the spherometer in place, and the imaginary segment, ABC_2C_1. Above is the equilateral triangle formed by the three feet C_1, C_2, C_3, while E at the center, marks the point where the radius BO, pierces the base of the segment. Then $BE = s$, is the sagitta, $EC_2 = d$, the distance from the point of the micrometer screw to the center of any leg, and $C_1O = BO = R$, the radius of the sphere of which the lens is a part. By geometry we have

$$BE\,(2R - BE) = \overline{EC_2}^2 \tag{251}$$

or

$$s\,(2R - s) = d^2 \tag{252}$$

whence

$$R = \frac{d^2 + s^2}{2s} \tag{253}$$

The distance d is usually measured once for all on the dividing engine or comparator, and is called the constant of the instrument. It may also be determined in terms of l, the length of one side of

the equilateral triangle formed by the feet of the spherometer as follows: Press the instrument firmly upon a piece of stiff paper until the positions of the three are left sharply defined. The length of the sides of the triangle may then be accurately measured by the vernier caliper. Then from Fig. 118,

$$d^2 - \frac{d^2}{4} = \frac{l^2}{4} \text{ or } d^2 = \frac{l^2}{3} \qquad (254)$$

whence by substitution

$$R = \frac{l^2}{6s} + \frac{s}{2} \qquad (255)$$

In practice the spherometer is first placed on a piece of plate glass and the zero reading accurately determined. It is then transferred to the lens and the readings upon the lens are made, care being taken to prevent the feet from slipping off the lens. The difference between the zero and the final readings gives the value of s. From the known value of d, the value of R is at once computed, or the value of l may be determined as shown above and the value of R computed from the equation (255).

FORM OF RECORD.

Exercise 80. To determine the radii of curvature of five lenses, by the spherometer.

$d = \ldots\ldots$ $l = \ldots\ldots$ Date.....

Zero	Lens 1	Lens 2	Lens 3	Lens 4	Lens 5
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
Mean	Mean	Mean	Mean	Mean	Mean
	s	s	s	s	s
	R	R	R	R	R

Measure both sides of each lens.

157. **Exercise 81. Radius of Curvature by Reflection.** The radius of curvature of a polished spherical surface may be determined by means of purely optical considerations if we employ the phenomena and formulae relating to spherical mirrors. Assume that the convex spherical surface mm' (Fig. 119), is

placed before the telescope T, at a distance A, and that it receives light from two brilliant objects L and L' symmetrically placed with respect to T. There will be formed in the mirror mm', two virtual, erect and diminished images of the objects L and L'. Owing to the inversion of these images by the telescope they are seen inverted in T. A small scale ss', placed in contact with the lens enables the observer to read off directly the apparent distance ss' between the two images. Now since the rays from L and L', after reflection at s and s' enter the telescope and seem to come from the images l and l', the normals Cs and Cs' will, if produced, bisect approximately the angles LsT and $L's'T$, and to the same degree of approximation, $PQ = 1/2LL'$ where P and Q are respectively the intersections on LL' of Cs and Cs' produced. Let $ss' = s$; OT, the distance from the lens to the objective of telescope, $= A$; $OC = R$ and $LL' = L$. Then from the triangles PQC and $ss'C$ we have

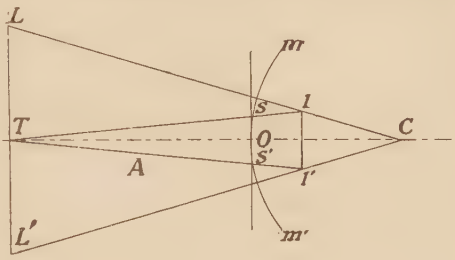

Fig. 119.

$$\frac{PQ}{ss'} = \frac{CT}{CO} \tag{256}$$

$$\frac{L/2}{s} = \frac{R+A}{R} \tag{257}$$

or

$$R = \frac{2As}{L-2s}. \tag{258}$$

In practice the lens is held in a clamp supported upon a tripod base, one foot of which bears an adjusting screw for tilting the lens about a horizontal axis. This foot should stand in a line parallel to the axis of the telescope, and normal to the lens surface. Two small lamps are placed at L and L' with their flames turned edge-wise to the lens. The telescope and lens are set up on two tables at least three meters apart, the lens facing the most

brightly lighted window in the room. The telescope is focused upon the lens surface until the scale ss' is sharply defined. One observer then takes one of the lamps and moves it slowly back and forth and up and down along the line TL, until the other catches sight of the moving image in the telescope.

It is to be noted that the image in a convex mirror is erect and is seen inverted owing to the inversion in the telescope; this inversion applies to the motions of the lamp as well, so that if the light moves to the right, the image seen in the telescope moves to the left and *vice versa*. Should the image fail to appear when the above directions are followed, the lens holder should be rotated slightly about its vertical axis until the image appears in the field. The image is then brought to the level of the scale by means of the adjusting screw in the foot of the lens holder. A black cloth placed close behind the lens renders the image much more bright and distinct.

Care must be taken to avoid confusion of the true images from the front surface of the lens, with the pair of erect images seen in the telescope which are due to reflection from the back of the lens; these images are originally inverted owing to the concave surface, and are erected by the telescope. Which pair of images must be chosen in case of a concave lens? What change is needed in the formula?

It will usually be found necessary to change the focus of the telescope very slightly in order to fix sharply the position of the image on the scale. This difference in focus becomes the more marked the more nearly the lens surface approaches a plane. The method is therefore best adapted to lenses of large curvature. The small scale may be dispensed with by pasting upon the lens two strips of paper with straight edges, parallel and facing each other. The perpendicular distance between the edges of the strips is then carefully measured with the vernier caliper and recorded. The lamps are then so adjusted that their respective images just disappear behind the edges of the paper. The measured distance is then equal to s. The distance A and L should be measured with a steel tape or a long stick and a metric rule.

Measure by this method the radii of curvature of three lenses.

FORM OF RECORD.

Exercise 81. To determine the radii of curvature of three lenses by the method of reflection.

Date.....

Lens No.	A	L	s	R
	.....	.....	.....	.....
	.....	.....	.....	.....
	.....	.....	.....	.....

158. Exercise 82. Focal Length of Lenses.[1] From the well known formula for the focal length of a lens

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad (259)$$

we may deduce an important relation under the condition that the object and image remain at a fixed distance, greater than $4f$, from each other. Let l be the distance between the object and the screen upon which the image is received. Then there will be two positions of the lens for which a sharp image is projected upon the screen, one near the object giving an enlarged image, with the lens at a distance p from the object and q from the image; and another nearer the screen giving a small but bright image. In this position the distances p and q are interchanged, so that now the lens is at a distance q from the object. Let a be the distance between these two positions of the lens. Then

$$p + q = l, \text{ and } q - p = a$$

whence

$$q = \frac{l + a}{2} \quad \text{and} \quad p = \frac{l - a}{2} \qquad (260)$$

substituting in (258) we have[2]

$$f = \frac{l^2 - a^2}{4l} \qquad (261)$$

The apparatus consists of an optical bench about two meters long, provided with a scale reading to millimeters and two sup-

[1] *College Physics*, Article 458.

[2] Owing to the fact that the distances p and q are not measured from the same point, but from the two principal points of the lens, this formula is not strictly accurate; the error is, however, not large. For the correction due to this approximation, see Glazebrook and Shaw, *Practical Physics*, p. 350.

ports to carry the screen and the lens. The object is placed at the zero end of the scale and at a suitable height above it so that the object, the center of the lens and the middle of the screen are all in the same straight line. In a thin board at the zero end of the scale is cut a hole 3 cm in diameter. This is closed by a piece of ground glass, which is strongly lighted by an incandescent bulb. A watch hand of elaborate design placed against the glass on the side toward the lens, forms a well defined, dark object upon a bright field.

A rough approximation to the value of f may be obtained by placing a piece of white paper in front of the object and bringing the lens toward it, until there is formed upon the paper an image of the window bars opposite, or of the trees and buildings outside. The reading of the lens carrier gives at once the **approximate focal length.** Why is not this the true value of f? The screen should then be placed at a distance from the object not less than five nor more than seven times this rough value of f. (Why?)

The lens is now shifted until a sharp image is projected upon the screen. The mean of five settings is taken as the position of the lens. The second position of the lens for a sharp image is then determined in the same way. The difference between these mean values is a, and this value with its related value of the setting of the screen l, will, when substituted in the formula, give a value for f. At least three different settings of the screen should be used and the mean of the three values of f returned as the focal length of the lens.

In the case of a concave lens, there can, of course, be no real image. Therefore, in order to use this method, it is necessary to combine the concave lens with a convex lens of suitable curvature, and determine first the focal length of the combination and then the focal length of the convex lens separately. If F be the focal length of the combination, and f' that of the auxiliary lens then the focal length of the concave lens f, is given by the relation

$$\frac{1}{f} = \frac{1}{f'} - \frac{1}{F}$$

or

$$f = \frac{F f'}{F - f'}. \qquad (262)$$

FORM OF RECORD.

Exercise 82. To determine the focal lengths of five different lenses.

Date.....

Lens No.	Screen (l)	Lens 1st position	Lens 2nd position	a	l^2-a^2	$4l$	f
.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....

Focal length of lens = ...

159. Exercise 83. Lens Curves. We have seen from Exercise 82 that for every setting of the screen there are in general two positions of the lens for which a sharp image is obtained. If now we plot the settings of the screen as ordinates and the corresponding settings of the lens as abscissae, we obtain what is known as the **lens curve,** Fig. 120 From our nomenclature the equation of the curve is

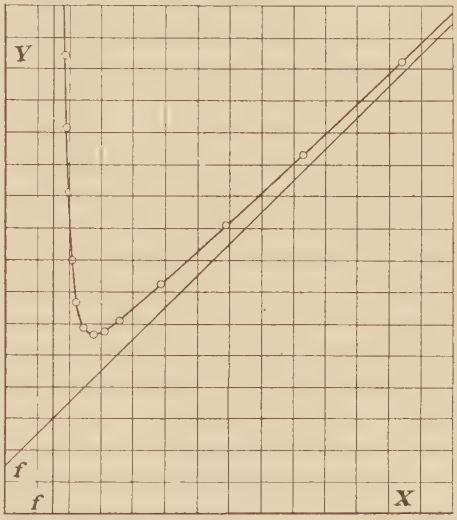

Fig. 120.

$$\frac{1}{x} + \frac{1}{y-x} = \frac{1}{f}. \qquad (263)$$

Is this the equation of an hyperbola? If so, what are its asymptotes? What is the physical interpretation of each? Use for this experiment a lens of about 15 cm focal length. Determine at least fifteen separate positions of the screen with their related settings of the lens. Plot the curve and draw the asymptotes.

Take pains to obtain as many as four or five points near the bend of the curve, *i. e.,* where the two images approach each other. What is the value of y for the lowest point of the curve? Determine the focal length of the lens from the curve.

FORM OF RECORD.

Exercise 83. Lens curves.

Lens..... Date.....

Screen (y)	Lens (x_1)	Lens (x_2)
.....	.....	.....
.....	.....	.....
.....	.....	.....

Focal length = ... Plot curve.

MAGNIFYING POWER.

160. **Exercise 84. Magnifying Power of the Telescope.** The telescope in its simplest form consists of two lenses, the object-glass or objective L, a convex lens of long focus, and the

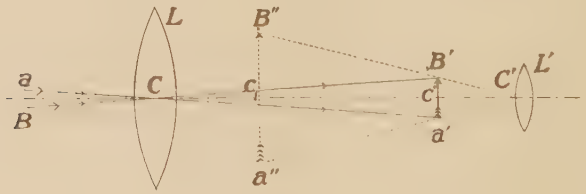

Fig. 121.

eye-piece L', a short focus lens either convex or concave. The distance from the object to the instrument is always great as compared with the focal length of the objective and the image is consequently smaller than the object in all cases. In case the eye-piece is a convex lens, (Fig. 121), this small image is viewed

directly by the eye-piece as an object placed nearer the lens than its focal distance. The result is a magnified virtual image of the image.

The effect of a telescope is to increase the visual angle subtended by a very distant object, that is, to bring an image of the object near the eye, so that when this image is viewed by the eye directly, the visual angle subtended by it is larger than that subtended by the object, in the ratio $F/25$, where F is the focal length of the objective and 25 cms represents the distance of distinct vision for the normal eye.[1] This relation is readily seen from Fig. 122, where the objective L forms an image of a distant object upon a screen. An eye at the center of the objective would see both image and object as of the same size, since the subtended angles are equal. If however, the eye approach the screen, the angle subtended by the image will increase until at a distance of twenty-five cms from the screen the image will appear larger than the object in the ratio $F/25$, as given above.

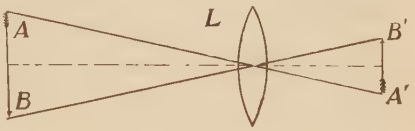

Fig. 122.

If the eye be brought nearer to the image in order to increase the magnification, its power must be increased by the use of a lens as a simple magnifier. Such a lens is termed an eye-piece. The magnification produced by the eye-piece is $25/f$ where the focal length of the eye-piece is f. The total magnification of the two lenses forming the telescope is therefore the product of the two, or F/f. In case the object is not at a great distance, F is no longer the focal length of the objective, but is the distance from the objective to the image formed by it. Consequently F/f changes with the distance of the object from the telescope. The ratio F/f is called the magnifying power of the telescope, and is most readily measured as follows:

A long scale is set up at one end of the room and so lighted that the divisions shall be seen sharp and clear. The telescope

[1] *College Physics*, Article 476.

is focused upon the scale so as to give a sharp image. The observer next looks through the telescope with the right eye and views the scale directly with the left eye. A little adjustment of the direction of the telescope and a little patience will enable the observer to see the two images formed by the two eyes, overlapping, so that he sees at the same time, (Fig. 123), the complete scale, and projected upon it, the magnified divisions of the scale image. By careful adjustment of the telescope the lengths of these magnified divisions may be read directly in terms of the divisions of the scale. Thus, suppose the half division from 4 to 4½ is seen projected upon the scale, its upper edge appearing to be at 4.10 and its lower edge at 8.15. It is clear that one half division seems to cover 4.05 divisions, hence the magnifying power is 8.10.

Fig. 123.

Care should be taken to avoid touching the telescope or its support during the measurements, as well as to avoid moving the head while comparing the upper and lower edges of the image for coincidence with the scale divisions. Measure the magnifying power of the telescope at distances of 4, 7, 10, and 15 meters from the scale. Next remove the field combination, by unscrewing the telescope at the first joint from the eye-piece, and taking out the lens found there. Repeat the measurement as above. What is the purpose of the field combination? How does the magnification vary with the distance?

FORM OF RECORD.

Exercise 84. To determine the magnifying power of a telescope.

Date.....

Distance from scale	Magnifying power with combination	Magnifying power without combination
.....	.....	.....
.....	.....	.....

Effect of distance upon magnifying power.

161. Exercise 85. Magnifying Power of Microscope. In a precisely similar way the magnification of a compound microscope may be measured by placing upon the stage of the instru-

ment an object micrometer, usually one containing a millimeter subdivided into tenths and hundredths, and focusing it sharply with good transmitted illumination. A millimeter scale held at the distance of distinct vision from the eye-piece is next placed in position and adjusted until, on looking through the microscope with both eyes open, the magnified image is seen projected upon the scale, and certain prominent lines of the two scales are made to coincide. The computation is identical with that in Exercise 84.

An alternative method is to place on the stage of the microscope the object micrometer, and by means of a camera lucida, or an Abbé illuminating prism, project the image of the scale directly into the eye-piece and view the two images with a single eye.

In the case of instruments provided with a micrometer eye-piece the magnification is determined by measuring with the micrometer the size of the image of known divisions on the object scale. The magnification is then the ratio between the size of image and the size of object.

Most microscopes are provided with a millimeter scale on the side of the draw tube. Measure the magnification of the microscope for at least three positions of the tube. How does the magnification vary with the position of the eye-piece?

FORM OF RECORD.

Exercise 85. To determine the magnifying power of a microscope.

Date.....

Microscope No.....
Object micrometer No.....

Reading on draw tube	Magnifying power
1	1
2	2
3	3

Effect of extending tube

INDEX OF REFRACTION.

162. **Exercise 86. Index of Refraction of Lenses from Radii of Curvature and Focal Lengths.** It is shown in geometrical optics that the focal length of a lens for any wave length is a function of the index of refraction μ, of the glass for light of

that wave length, and of the radii of curvature of the lens. This relation is given by the equation[1]

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \qquad (264)$$

If we have the values of f, r_1 and r_2 for any lens we may compute the index of refraction at once from the above formula. The focal length having been obtained by means of white light, the resulting value of μ will of course refer to no definite color, but will in general correspond to the brightest part of the spectrum, *i. e.*, to the part between the lines D and E.

FORM OF RECORD.

Exercise 86. From the values of f, r_1 and r_2 for the lenses measured in exercises 80 and 82, compute the mean index of refraction for each lens.

Date.....

Lens	r_1	r_2	f	μ
1	.....	.....	.....	.....
2	.....	.....	.....	.....

How may equation (264) be simplified, when one of the radii is infinite? When the two radii are equal?

163. **Exercise 87. Index of Refraction by Means of a Microscope.** It is shown in works on physics[2] that if a point A, (Fig. 124), be viewed vertically through a **transparent plate of thickness** OA and refractive index μ, the point will **appear to be raised** to some position I in the vertical, such that $OA = \mu\, OI$, or

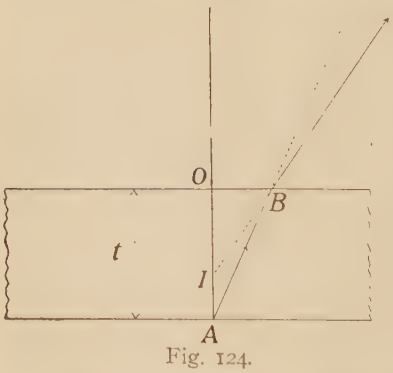

Fig. 124.

$$\mu = \frac{OA}{OI} = \frac{t}{t - a}. \qquad (265)$$

where $AI = a$.

In this way the **index of refraction** of a **transparent plate**, or of a **layer of fluid** may be determined by means of a microscope furnished with a scale and vernier on its tube.

[1] For the derivation of this formula and its interpretation see *College Physics*, Articles 454-457.

[2] *College Physics*, Article 448.

In practice the microscope, fitted with a low power objective is focused upon a mark on a piece of stiff paper, or better upon a scratch in a piece of flat metal, held upon the microscope stage by means of clips or bits of wax. The instrument having been sharply focused upon some prominent feature of the scratch, the position is taken by reading the scale and vernier on the tube. The transparent plate, usually a plate of glass some 5 mm thick, is next placed upon the stage **above and immediately in contact with** the scratch in the plate. The microscope is again focused upon the same feature of the scratch **through** the plate, and the reading taken as before. The microscope is then focused upon the upper surface of the plate and the reading made. From these three readings, each being the mean of at least five separate settings, the values of OA and OI are readily determined and the value of μ computed from the formula.

For **liquids** a small flat-bottomed dish is fastened to the microscope stage by two bits of wax, and the instrument focused upon a scratch on the upper surface of the bottom. The liquid is added by means of a medicine dropper to a depth of from 3 to 5 mm, and the reading taken upon the **same scratch through the liquid.** A few grains of lycopodium powder are then sifted upon the surface of the liquid, the microscope focused upon a grain of the floating powder and the reading taken as before. For liquids the instrument must of course stand vertical. In case the readings differ by as much as 0.06 mm, the mean of a larger number of readings must be taken. The depth of the liquid may be increased after each determination, and readings **through** the liquid and **on top** of the liquid give data for a new value of μ.

Determine by this method the refractive indices of two pieces of glass and of distilled water.

FORM OF RECORD.

Exercise 87. To determine the indices of refraction of glass and of distilled water by means of a microscope.

Date.....

Reading on scratch	Through subst.	On top	t	$t-a$	μ
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....

THE SPECTROMETER.

164. Description. A spectrometer consists essentially of an achromatic telescope, a graduated circle and a collimator, or telescope with the eye-piece replaced by an adjustable slit, for producing a beam of parallel rays. The telescope is mounted upon a substantial tripod base, so as to move freely about the axis of rotation of the instrument. The collimator is usually fixed in position, but has slight freedom of movement for purposes of adjustment. The graduated circle may or may not rotate about the axis of the instrument, but it must in any case be capable of being firmly clamped at will, and be provided with verniers or reading microscopes to determine the position of the telescope at any time.

In the Geneva Society instrument shown in Fig. 125, the prism table is provided with a radial arm carrying a vernier along the graduated circle, by means of which the rotation of the table may be accurately determined. By means of a set screw the table may be clamped to this arm or released from it, and when free it may be raised or lowered, or rotated at will about its own axis for purposes of adjustment. It is also provided with three levelling screws, symmetrically placed about its axis for bringing the surface of prism or grating parallel with the axis of rotation.

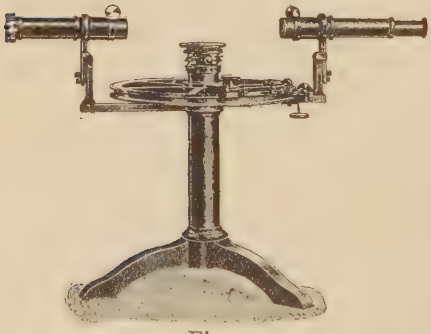

Fig. 125.

Both the radial arm and the arm carrying the telescope are provided with clamp and slow motion screws for accurate setting upon any desired feature in the field of view. Through adjusting screws attached to their supports, both telescope and collimator are capable of slight motion in a vertical plane in order to bring their optical axes accurately perpendicular to the axis of rotation of the instrument.

The telescope is also provided with a **Gauss eye-piece**, for

illumination of the cross-hairs in making certain adjustments and measurements. This eye-piece (Fig. 126), has an opening in one side and a piece of transparent, plane parallel glass set across the tube at an angle of 45° to the axis, by which light entering the opening is reflected parallel to the optical axis of the telescope. When the cross-hairs are brought into the focal plane of the objective, light passing them leaves the objective as parallel rays and after striking a plane surface placed normal to the axis of the telescope, is reflected directly back into the telescope forming in its focal plane a dark image of the cross-hairs themselves.

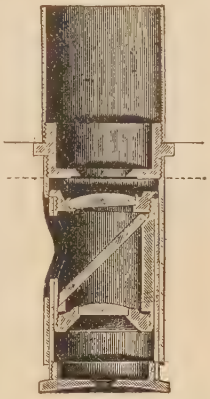

Fig. 126.

A far more delicate and useful device is the **Zeiss-Abbé eye-piece,** (Fig. 127), in which a small, **totally reflecting prism** is inserted in the focal plane of the telescope at one side of the field of view, and provided with an adjustable slit on the side toward the objective. The slit and cross-hairs lie in the same plane, and when the telescope is focused for infinity, light entering through a small window in the side of the eye-piece is reflected by the prism through the slit and emerges from the objective as parallel rays. A plane surface placed normal to the axis of the telescope returns the light as from an infinite distance and shows a sharp image of the slit in the focal plane of the telescope. By this arrangement the functions of collimator and observing telescope are combined, the reflected image is sharp and unmistakable, and the system offers many important optical advantages, some of which will be mentioned later.

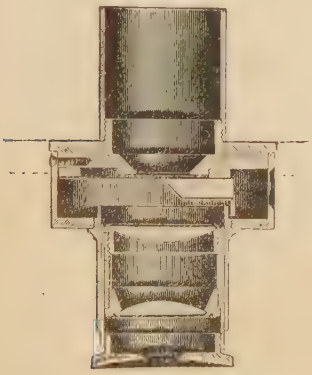

Fig. 127.

165. **Adjustments of the Spectrometer.** Before the spectrometer is ready for use a number of adjustments must be made,

some of which must be repeated at frequent intervals, while others should be needed but rarely. It is not expected that the beginner should attempt such adjustments for himself, but rather that he may obtain an intelligent idea of the working of the instrument from a careful study of them. The adjustments are given in the order that they should be made by an observer on beginning work upon a new instrument.

(*a*) *The cross-hairs.* The eye-piece has at its focus a pair of fine hairs, termed cross-hairs, which must be sharply seen by the eye on looking into the telescope. If the cross-hairs are not sharp, the eye-piece must either be drawn out or pushed in with a gentle twisting motion until they are seen sharply defined on a white field when the telescope is turned toward the window. This is the first adjustment to be made in every case, and as the adjustment is slightly different for different persons, it must be made each time before any work is attempted with the telescope.

(*b*) *The telescope.* The telescope must be focused for parallel rays. In instruments provided with a Zeiss-Abbé eye-piece this adjustment is readily effected by placing in front of the objective a piece of glass with a good plain surface; a piece of French plate mirror glass will do very well. After the slit is illuminated the telescope is focused upon the reflected image until it is seen clearly defined. The telescope is then in adjustment. Although the same end may be accomplished by use of the Gauss eye-piece, the result is not so satisfactory since the images are always faint and lacking in distinctive features by which to judge the accuracy of focus. A small dust particle adhering to the hairs may sometimes furnish the desired criterion.

(*c*) *The collimator.* The telescope is next turned so as to look directly into the collimator. A small lamp is placed behind the slit, which is slightly opened, and the outer end of the collimator either pulled out or pushed in until the slit is seen sharply focused in the telescope. The images of slit and cross-hairs must show no parallax, that is, there must be no apparent motion of slit and cross-hairs with reference to each other as the head is slightly moved from side to side while looking into the telescope. The collimator is now in adjustment, since the telescope focused

for parallel rays shows the slit sharply defined, and the rays emerging from the collimator objective must therefore be parallel.

It is well to mark this position of the collimator tube in order that it may be replaced with little trouble in case it should be accidentally displaced. In some instruments the collimator tube may be clamped in position, once it is adjusted for focus, but this arrangement is neither common nor necessary. For laboratory work under ordinary conditions it is usually more expedient to focus and adust the collimator beforehand and require the student to confine his manipulations to the telescope and prism.

(*d*) *The optic axes of telescope and collimator.* 1. The **axes** of **telescope** and **collimator** must be **perpendicular** to the **axis** of **rotation** of the instrument. This adjustment is most readily secured by the use of one of the collimating eye-pieces mentioned above, together with a small plate of plane parallel glass silvered on both sides.

If the Gauss eye-piece be employed, it is inserted in the telescope and focused upon the cross-hairs, care being taken to leave the glass reflector as nearly vertical as possible. A small lamp on an adjustable support is then brought up to within a few inches of the opening and so adjusted both laterally and vertically that on placing a square of good mirror glass over the objective the image of the diaphragm carrying the cross-hairs is seen filled with light, and on focusing the telescope slightly the cross-hairs are seen sharply defined on a bright field. A slight tilting of the mirror glass will show by the disappearance of the circle of light that the illuminated circle is due to reflection from it and not from the posterior surfaces of either of the two intervening lenses.

The small plate of plane parallel glass is then mounted upon the table of the spectrometer, either upon an adjustable table of its own, or upon two pieces of soft wax, so as to stand as nearly vertical as may be, and with its face parallel to a line connecting two of the three levelling screws of the prism table. It can then be rotated slightly about this line by means of the third screw. The plate is then turned so as to throw into the telescope the light issuing from the objective, and adjusted until the image of the cross-hairs is again in the field and brought into coincidence with

the hairs themselves. If on rotating the plate through 180° the images are again in coincidence the plate stands parallel to the axis of rotation, and the axis of the telescope is normal to it. If this be not the case then the reflected image from the second side must be brought into coincidence with the cross-hairs, by altering by equal amounts the positions of the plate and telescope. On reversing the plate a similar procedure will soon give the desired adjustment. Finally the telescope is turned to face the collimator, the slit of which has been illuminated and turned horizontal. The level of the collimator is then adjusted until the image of the slit coincides with the horizontal cross-hair of the telescope. If the parallel plate be now turned to reflect the image into the telescope, the slit image should remain on the horizontal hair as the telescope follows it around the circle.

2. The **optic axes must intersect** the **axis of rotation** of the instrument. This adjustment in many instruments is cared for by the maker, and the telescope and collimator are incapable of motion about a vertical axis. In others the two tubes are adjustable about a vertical axis, and are very liable to be displaced. The adjustment is most readily secured by placing over the objective of the telescope a cap containing a lens whose focal length is slightly less than the distance from objective to the center of the table. A bullet suspended by a cocoon fiber is then brought directly over the center of the table by means of a suitable support, and the telescope turned about its vertical axis until the image of the thread falls upon the intersection of the cross-hairs. Its vertical axis is then firmly clamped, the lens removed, and the telescope swung round facing the collimator, which is then adjusted till the slit image likewise falls in the center of the field, when the collimator is also firmly clamped.

The adjustment should not be disturbed except for special reason, and the student when rotating the telescope about the circle, should **take hold close up to the circle**, and **not by the outer end**.

166. **Reflecting Surfaces.** A reflecting surface intended for optical work should be accurately ground and polished. Many surfaces polished on cloth show **innumerable fine ripples** when

examined in good light, at a distance of about 15 inches from the eye. Such surfaces are worthless for the formation of optical images.

Surfaces of prisms should be **true planes**, standing **normal to the base** of the prism and should **extend up to the edges of the prism.** Many otherwise good prismatic surfaces are rounded off at the edges by careless work in polishing. A **perfect** prism **surface** should show **no curvature at the edges** when tested by interference fringes upon a true plane.

It is frequently desirable to examine a reflecting surface in order to see whether or not it is clean, unbroken or properly illuminated. In many cases the surface may be inaccessible or at a temperature which renders close inspection impossible. In such cases a telescope showing an image formed by light reflected from the surface in question affords the means for such examination. A short focus lens, as a common reading glass, held between the telescope and the eye and properly focused, gives a well defined image of the prism surface, and enables the operator to examine it at leisure. This is especially useful in work with prismatic and interferometer surfaces, as the exact condition of the reflecting surface and the point from which the reflected light proceeds can be instantly and accurately determined.

In adjusting the faces of a prism for work on the spectrometer, one face is set at right angles to the line joining two of the three levelling screws of the table, in order that its adjustment may be undisturbed by the motion of the third screw. In the case of a prism whose angles are each about sixty degrees, and on a table whose levelling screws are placed symmetrically about the axis, this arrangement may be realized for all three faces.

In some prisms one face may not be exactly perpendicular to the base of the prism, and the image of the slit as reflected from this side is slightly inclined to the vertical cross-hair. In such cases it is impossible to have all the faces parallel to the axis of revolution at the same time. This fault is slightly noticeable in most prisms, but by bringing the image of the slit to the middle of the field in each case and setting the cross-hair upon the middle of the image, the error may be rendered practically negligible.

167. **Exercise 88. To Measure the Angle of a Prism.** In measuring the angle of a prism with the spectrometer shown in Fig. 125, three methods of procedure are possible. Each method will be described in turn.

(*a*) *Prism fixed and telescope rotated.* The spectrometer having been put in adjustment the prism is placed upon the table as directed in Article 166, and its faces set approximately vertical by the eye. The collimator slit is opened rather wide and illuminated by a small lamp. The angle to be measured is turned toward the collimator and so placed as to divide the opening of the objective about equally. In the case of telescopes of small aperture it is important also that the angle be placed very near the axis of rotation, as otherwise the reflected rays may be thrown out of the field of view.

Turn the telescope to position *T*, (Fig. 128). A black cloth placed loosely over the collimator, prism and telescope aids materially in finding the reflected image of the slit in the telescope. It is best to **catch the reflected image first in the eye** placed close up to the prism and then, keeping the image in view slowly bring the telescope into position. Having brought the right hand image into the telescope observe whether or not it lies in the center of the field and parallel to the vertical hair. If not, the prism must be adjusted for level. Next turn the telescope to position *T'*, and see whether the left hand image is also visible, whether it lies properly in the field, and whether the telescope is in such a position that readings may be made in each case. .

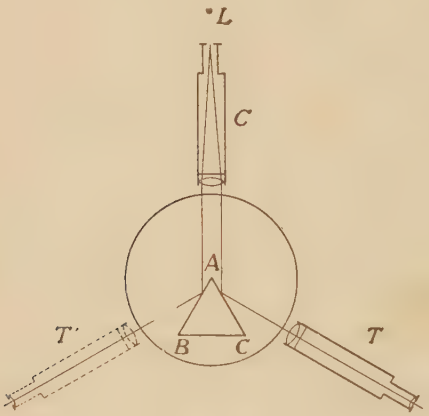

Fig. 128.

If the prism need adjustment for level, observe carefully which face is most out, and note the effect upon the position of the

images when the prism is slightly rocked about each edge forming the angle in question. If the prism has been properly placed upon the table a few minutes trial should bring it into adjustment. In case the faces are not both normal to the base, the images cannot both be rendered vertical. In such case set the intersection of the cross-wires on the center of the image.

The slit image is next drawn down to a narrow line by means of the adjusting screw on the collimator, and brought upon the vertical hair by use of the clamp and slow motion screws. Owing to the fact that the eye can better judge of the equality of the two bright parts of the slit on either side of the dark hair, it is preferable not to make the slit narrower than from three to five times the width of the hair. The setting having been completed the position of the telescope is read and recorded. If the instrument have two verniers or two microscopes read both each time and combine the readings as indicated in Article 28. Unclamp the telescope and turn to position T' and repeat the operations just described. Then return to position T, set and read, to make sure that nothing has been changed. Combine this reading with the first made. The difference between this mean and the reading at T' is twice the angle A, of the prism. Prove this. Displace the prism slightly and repeat the measurements twice. Take the mean of the three as the final result.

(*b*) *Telescope fixed, prism rotated.* Turn the telescope (Fig. 129), so as to make an angle of 20° to 30° with the collimator and clamp it. Rotate the prism table and adjust until the reflected image from face AC enters the telescope and lies properly in the field. Then continue the rotation until the image from face AB also enters the field. If both images are properly located for measurement, clamp the table to its radial arm, after making sure that the vernier can be read in each position. Bring the image from the first face once more into the field and clamp the arm, making the final setting with the slow motion screw. Record the reading. Unclamp the arm, rotate

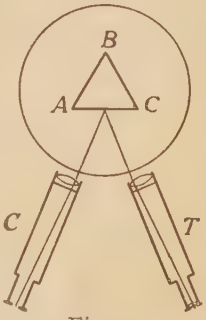

Fig. 129.

prism to second position, set and read. The angle through which the prism has been rotated is $180° - A$. Take three sets of readings as under (a), unclamping the table from the arm and displacing slightly so as to change the readings in each case. Take the mean of the three results.

(c) *By autocollimation.* The prism is placed centrally over the axis of the table and the telescope, fitted with either of the collimating eye-pieces described in Article 166, is clamped in position. The illumination from the side is adjusted and tested by the small mirror placed over the objective. The prism is then turned so that face AC (Fig. 130), stands approximately normal to the axis of the telescope and slowly rotated until the bright image either of the diaphragm or slit enters the field. The prism is then adjusted until the reflected images from the two sides register accurately upon the cross-hairs. The table is then clamped to the arm and readings made upon the two faces in succession. The angle through which the prism has been displaced is again $180° - A$. Make three independent sets of readings, displacing the table relatively to the arm after each set as under (b).

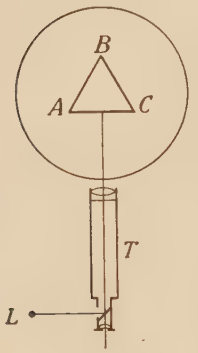

Fig. 130.

FORM OF RECORD.

Exercise 88. To measure the angle of a prism by three different methods.

Date.....

	1st Face	2nd Face	Difference
(a) Vernier I			
" " II			
Mean.....	.....	.....	.....
		Angle.....	

(b) Similar record.
(c) " "

Angle by method (a) =
" " " (b) =
" " " (c) =
Mean =

168. **Angles by Method of Repetition.** In work of precision it is frequently of advantage to employ the principle of repetition

in the measurement of angles. With a spectrometer in which the movement of telescope and of prism table may be read separately, or in which the prism table may move either with, or independently of, the graduated circle, the principle may be employed very readily in either method *b* or *c*, as described above.

For example suppose that in method (*b*), the prism has been rotated from the first to the second position by means of the radial arm, and the readings determined in each case. If now leaving the arm fixed, the table be released and the prism rotated back to its first position, the setting made, and the table again clamped to the arm, and arm and table again rotated in the same direction as at first until the slit image from the second face again falls upon the cross-hairs, it is clear that the difference between the initial and final reading of the arm vernier will correspond to a total rotation of 2 $(180° - A)$ or $360° - 2A$. If the operation be repeated once more the displacement becomes $540° - 3A$, or, in case A is about 60°, the total displacement is about 360° and the initial and final readings are made upon the same part of the circle.

The method therefore consists in stepping off the angle a sufficient number of times to bring the index back to the neighborhood of the starting point, where the graduations are assumed to differ little from each other. It is designed to eliminate errors of graduation in the circle, and assumes that these errors are greater than those of individual settings. The method is employed to the greatest advantage when the angle to be measured is some aliquot part of 360°.

In some instruments the graduated circle is read from fixed verniers or microscopes, and may move either in conjunction with the telescope or each may move independently. In such instruments the principle may be applied to method (*a*). Thus suppose the reading has been made in position T; the telescope is clamped to the circle and the two rotated into position T', the prism remaining fixed. The angle thus stepped off is $2A$. The circle is now clamped and the telescope released, returned to position T, accurately set and clamped to the circle, which is now re-

leased and telescope and circle again moved forward through the angle $2A$.[1]

169. **Exercise 89. Index of Refraction of a Glass Prism.** The effect of a prism upon light passing through it is two-fold. The direction of the light is changed, the light being bent toward the base of the prism both on entering and leaving the prism, and secondly, the light is dispersed or broken up into its constituent colors. If the prism used in the previous exercise be now placed centrally over the center of the table and turned into the position indicated by the full line (Fig. 131), an eye, placed in the position indicated by the emergent light, will perceive no longer a bright image of the slit, but a broad band of color, the spectrum of the light furnished by the lamp. This spectrum may now be received into the telescope and its parts examined. The best effect is obtained by excluding all stray light from the telescope by means of the dark cloth as in Exercise 88. By rotating the prism slowly and following the spectrum with the telescope, a position is soon found in which, no matter which way the prism is rotated, the spectrum comes to a certain point nearest the direct line from the collimator, stops and then recedes. This is the **position of minimum deviation.** The small lamp is now removed and a Bunsen burner substituted. The burner is so arranged that the colorless flame plays against the tip of a piece of asbestos paper saturated with sodium nitrate. An intense yellow light results. On examining the image in the telescope it is seen that the spectrum of this light consists of a single bright line, a yellow image of the slit, the sodium spectrum, **for this temperature.** In spectroscopes of high resolving power this line is readily seen to consist of two lines, D_1 and D_2.

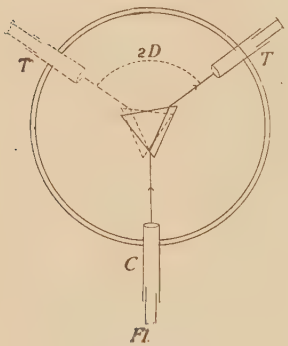

Fig. 131.

[1] See Louis Bell, *The Absolute Wave Length of Light, Am. Jour. Sci.*, XXXV, pp. 350-352.

By closing the openings of the Bunsen burner so as to give the luminous flame, the continuous spectrum returns and we see superposed upon it the bright line due to the vapor of incandescent sodium. The non-luminous flame having been restored, the prism is rotated until the position of minimum deviation for the sodium line is actually determined, the cross-hair placed upon the image of the slit, the telescope clamped and the reading taken.

The prism is next rotated into the position shown by the dotted line in the figure. The light is now deviated to the left of the direct position and the position of minimum deviation is determined as before. The difference between the two readings is obviously $2D$, where D is the angle of **minimum deviation for sodium light.** It is shown in works on physics,[1] that when the prism is put in the position of minimum deviation, the refractive index μ, is defined by the equation

$$\mu = \frac{\sin \tfrac{1}{2}(A + D)}{\sin \tfrac{1}{2} A} \qquad (266)$$

where A is the angle of the prism. Derive this formula.

From the measured values of D and A as obtained above, compute the value of μ for sodium light for the prism under experiment. Repeat the experiment using lithium carbonate in place of sodium nitrate.

FORM OF RECORD.

Exercise 89. To determine the index of refraction of a prism for the lines Na_α and Li_α.

Prism..... Date.....

T	T'	$2D$	D
.....	.....	.....	.....
.....	.....	.....	.....

A .....
D .....
$A + D$ .....
$\tfrac{1}{2}(A + D)$ .....
$\tfrac{1}{2} A$ .....

Computation

$\log \sin \tfrac{1}{2}(A + D)$ .....
$\log \sin \tfrac{1}{2} A$ .....
$\log \mu$ .....
μ .....

[1] *College Physics*, Article 450.

DIFFRACTION.

170. Exercise 90. Wave-length of Sodium Light by Diffraction Grating. A Bunsen burner (Fig. 132), is provided with a sheet iron hood in which is cut a small triangular slit, s, about 1.5 cm long. Immediately in front and below the slit is fixed a meter rod held horizonally with the slit at the center of the rod. At a distance of some three or four meters in front of the slit is placed the grating, held in a suitable clamp, with its surface vertical and parallel to the meter rod. If the burner be adjusted for the luminous flame the slit appears white to the naked eye but when viewed through the grating the eye perceives in addition

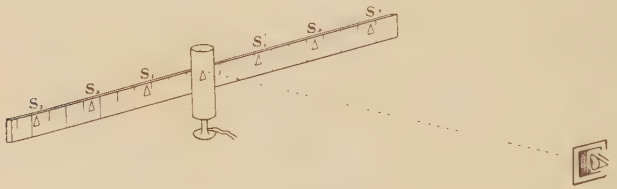

Fig. 132.

to the white slit, a number of spectra symmetrically placed with reference to the central image. These are diffraction spectra and are characterized (a) by the relative positions of the various colors with respect to the slit, the violet being the least diffracted and the red the most; (b) by the uniformity of the dispersion of the various spectra, each color being seen at a distance from the slit directly proportional to the wave-length of the light in question.

In practice the luminous flame is replaced by the sodium light and the colored spectra become a series of yellow images of the slit which, to an eye placed behind the grating, are seen projected upon the meter rod at spaces equidistant from the central image. Beginning at the inner spectra measure carefully the distances $s_1s'_1$, between the first two images on either side of the slit, $s_2s'_2$, the distance between the next two, and so on. Take half the measured distance as the distance of each image from the central slit, ss_1, ss_2, and so on.

If d be the grating space, n the order of the spectrum observed, and θ_n the angle subtended at the eye by the distance ss_n, then[1]

$$n\lambda = d \sin \theta_n \qquad (267)$$

where λ is the wave-length of sodium light. Hence for the first three or four spectra,

$$\lambda = d \sin \theta_1 = \frac{d \sin \theta_2}{2} = \frac{d \sin \theta_3}{3}, \text{ etc.} \qquad (268)$$

In the experiment described the distances ss_n divided by a, the distance from the rod to the grating, gave directly $\tan \theta_n$, in each case, from which the value of $\sin \theta_n$ is readily found.

Determine by this method the wave-length of sodium light, using spectra of at least four different orders. The value of d for the grating used will be given by the instructor. Calculate λ in millimeters.

FORM OF RECORD.

Exercise 90. To measure the wave-length of sodium light by diffraction grating.

$d = \ldots\ldots$ $s = \ldots\ldots$ $a = \ldots\ldots$ Date.....

s_n	s'_n	$\frac{s_n s'_n}{2}$	$\frac{ss_n}{a}$	$\frac{\sin \theta_n}{n}$	λ

171. Exercise 91. Constant of a Diffraction Grating. If in equation (267) the wave-length of light be assumed as known, the quantities n and $\sin \theta_n$ may be determined and d the grating constant may be computed. Place a transmission diffraction grating, whose constant is to be determined, centrally upon the table of the spectrometer and set its surface normal to the axis of the collimator. This is most readily done by setting the telescope upon the image of the illuminated collimator slit, and then set-

[1] *College Physics*, Article 494.

ting the **grating normal** to the **axis** of the **telescope,** by means of an **auto-collimating eye-piece.**

Illuminate the slit with sodium light, and determine θ for the first two spectra on either side the central image. Assume for λ its mean value 0.0005893 mm, and compute the value of d.

FORM OF RECORD.

Exercise 91. To determine the constant of a diffraction grating by the spectrometer.

Grating used..... Date.....

Readings	Right	Left	
First spectrum			$2\theta_1 =$ $\theta_1 =$ $d =$
Second "			$2\theta_2 =$ $\theta_2 =$ $d =$
			Mean

172. Dispersion, Normal and Prismatic. We have seen in Exercise 90, that the spectrum of a diffraction grating is characterized by the uniformity of the dispersion produced, the deviation of each color being absolutely fixed by the equation connecting the wave-length of the color in question and the constant of the grating. Spectra formed by two diffraction gratings are **directly comparable,** the ratio of **their lengths** being **inversely** as the **grating constants.** Such spectra are termed **normal spectra,** and such dispersion, **normal dispersion.**

With prisms however the case is very different. If we examine the spectra from a number of prisms of different materials, but all having the same refracting angle, we shall find that the lengths of the spectra differ enormously. This is said to be due to the different **dispersive powers** of the different prismatic substances. Again if the angles of the prisms be so adjusted that the resulting spectra are all of the same length, we shall still find that the separation of the colors in different parts of the spectrum is very different in the different substances. **There is no definite relation** therefore **between** the **change of index of refraction and change of wave-length,** the **dispersion** in each case **depending** upon the **nature** of the **refracting substance.**

This peculiarity is termed **irrationality of dispersion.**

173. **Exercise 92. Dispersion Curve for a Prism.** Owing to the irrationality of dispersion, the spectra from prisms of different material are not directly comparable with each other and it becomes necessary to investigate experimentally the relation existing between wave-length and index of refraction for any given prism before it can be used as an instrument for study of unknown lines in the spectrum.

This is most readily done by determining the indices of refraction of the prism for a number of prominent well known lines in different parts of the spectrum, and plotting on co-ordinate paper the indices as ordinates and the corresponding wave-lengths as abscissæ. Through the points thus determined a smooth curve is drawn which is termed **the dispersion curve,** for the prism in question.

By means of this curve the wave-length of an unknown line may be determined as soon as its index of refraction with the given prism is known, or the index for a line of given wave-length may be predicted. The curve may be represented with a fair degree of accuracy by Cauchy's dispersion formula

$$\mu = A + \frac{B}{\lambda^2} \qquad (269)$$

where A and B are constants depending upon the nature of the prism substance. From a series of related values of λ and μ, we may determine the constants A and B.

Determine according to Exercise 89, the indices of refraction of a given prism for the bright lines K_α, K_β, Li_α, Na_α, Tl, Sr_δ, and plot a curve with the indices thus obtained as ordinates, and the corresponding wave-lengths, (Table XIII), as abscissæ. Take off from the curve the indices of the prism for the two hydrogen lines H_α and H_β, corresponding to the C and F lines in the solar spectrum using the values for λ given in Table XIII. Using sunlight verify the indices by actual measurement. Determine indices to three decimals.

FORM OF RECORD.

Exercise 92. To construct dispersion curve for Prism No..... and determine from it the indices of refraction for two lines of given wave-length.

Date.....

Line	D	μ	λ	
K_α				
Li_α				
Na_α				
Tl				
Sr_δ				
K_β				

	From curve	Observed	Difference
μ for Hα			
" Hβ			

TABLES.

TABLE I.—*Atomic Weights of Some Elements.*

	Hydrogen = 1	Oxygen = 16.0		Hydrogen = 1	Oxygen = 16.0
Aluminum	26.90	27.1	Nitrogen	13.93	14.04
Cadmium	111.55	112.4	Platinum	193.30	194.80
Chlorine	35.18	35.4	Potassium	38.85	39.15
Copper	63.12	63.6	Silver	107.11	107.93
Gold	195.70	197.2	Sodium	22.88	23.05
Lead	205.35	206.9	Sulphur	31.82	32.06
Mercury	198.50	200.0	Tin	118.10	119.00
Nickel	58.30	58.7	Zinc	64.91	65.40

TABLE II.—*Density of Water at Different Temperatures.*

t	*d*	*t*	*d*	*t*	*d*
0°	0.99988	11°	0.99965	21°	0.99806
1	0.99993	12	0.99955	22	0.99784
2	0.99997	13	0.99943	23	0.99761
3	0.99999	14	0.99930	24	0 99738
4	1.00000	15	0.99915	25	0.99713
5	0.99999	16	0.99900	26	0.99688
6	0.99997	17	0.99884	27	0.99661
7	0.99993	18	0.99866	28	0.99634
8	0.99988	19	0.99847	29	0.99606
9	0.99982	20	0.99827	30	0.99577
10	0.99974				

TABLE III.—*Density of Mercury at Different Temperatures.*

t	*d*	*t*	*d*	*t*	*d*
0°	13.5950	11°	13.5679	21°	13.5433
1	.5925	12	.5654	22	.5408
2	.5901	13	.5629	23	.5384
3	.5876	14	.5605	24	.5359
4	.5851	15	.5581	25	.5335
5	.5827	16	.5556	26	.5310
6	.5802	17	.5531	27	.5286
7	.5777	18	.5507	28	.5261
8	.5753	19	.5482	29	.5237
9	.5728	20	.5457	30	.5212
10	.5703				

TABLE IV.—*Densities of Various Bodies.*

Alcohol at 20° C.	0.789	Iron, wrought	7.86
Aluminium	2.58	Lead	11.3
Brass (about)	8.5	Mercury at 0° C.	13.595
Brick	2.1	Nickel	8.9
Copper	8.92	Oak	0.8
Cork	0.24	Paraffin	0.90
Diamond	3.52	Pine	0.5
Glass, common	2.6	Platinum	21.50
" heavy flint	3.7	Quartz	2.65
Gold	19.3	Silver	10.53
Ice at 0° C	0.916	Tin	7.29
Iron, cast	7.4	Zinc	7.15

TABLE V.—*Reduction of Barometer Readings to 0°C.*

When the height of a mercury column has been measured with a brass scale, the length of which is correct at 0°C., the temperature of the barometer being t°C., the following quantity, corresponding to temperature and height, has to be subtracted from the reading. (See equation 35.)

	Observed Height of Barometer.					
t	72.0 cm.	73.0 cm.	74.0 cm.	75.0 cm.	76.0 cm.	77.0 cm.
10°	0.117 cm.	0.118 cm.	0.120 cm.	0.122 cm.	0.123 cm.	0.125 cm.
11	.128	.130	.132	.134	.135	.137
12	.140	.142	.144	.146	.148	.150
13	.152	.154	.156	.159	.160	.162
14	.163	.166	.168	.170	.172	.175
15	.175	.177	.180	.182	.185	.187
16	.187	.189	.192	.194	.197	.200
17	.198	.201	.204	.207	.209	.212
18	.210	.213	.216	.219	.222	.225
19	.222	.225	.228	.231	.234	.237
20	.233	.237	.240	.243	.246	.249
21	.245	.248	.252	.255	.259	.262
22	.257	.260	.264	.267	.271	.274
23	.268	.272	.276	.279	.283	.287
24	.280	.284	.288	.292	.295	.299
25	.292	.296	.300	.304	.308	.312
26	.303	.307	.312	.316	.320	.324
27	.315	.319	.324	.328	.332	.337
28	.327	.331	.336	.340	.345	.349
29	.338	.343	.348	.352	.357	.362
30	.350	.355	.360	.365	.369	.374

TABLE VI.—*Coefficients of Elasticity.*

Substance	Volume Elasticity e	Simple Rigidity n	Young's Modulus M	Velocity of Sound
	10^{11} dynes per cm²	10^{11} dynes per cm²	10^{11} dynes per cm²	10^5 cms. per sec.
Distilled water....	0.222	.....	.....	1.45
Glass (flint) ...	3.47 to 4.15	2.35 to 2.40	5.74 to 6 03	5.0
Brass.....	10.02 to 10.85	3.44 to 4.03	9.48 to 11 2	3.4
Steel.....	18.41	8.19	20.2 to 24.5	5.1
Iron (wroug't)	14.56	7.69	19.63	5 1
Iron (cast).	9.64	5.32	13.49	5.0
Copper....	16.84	4.40 to 4.47	11.72 to 12.34	3.7

TABLE VII.—*Viscosity and Surface Tension of Liquids at 20°C.*

	Coeff. of Viscosity	Surface Tension
	grams per cm. sec.	dynes per cm.
Alcohol	0.0125	23
Ether	0.0025	18
Glycerine	8.5	
Mercury	0.015	68
Machine Oil	1.95	
Olive Oil	0.225	35
Turpentine	0.0015	27
Water	0.0102	73

TABLE VIII.—*Moments of Inertia.*

Uniform thin Rod, axis through middle, length $= l$	$I = M\frac{l^2}{12}$
Rectangular Lamina, axis through center and *parallel* to one side, a and b length of sides, a the side bisected	$I = M\frac{a^2}{12}$
Rectangular Lamina, axis through center and *perpendicular* to the plane, a and b length of sides	$I = M\frac{a^2+b^2}{12}$
Rectangular Parallelopiped, axis through center and *perpendicular* to a side; a, b and c length of sides, axis perpendicular to side contained by a and b	$I = M\frac{a^2+b^2}{12}$
Circular Plate, axis through center perpendicular to the plate, radius $= r$	$I = M\frac{r^2}{2}$
Circular Ring, axis through center perpendicular to plane of ring, outer radius $= R$, inner radius $= r$	$I = M\frac{R^2+r^2}{2}$
Right Cylinder, axis the axis of figure, r = radius of section	$I = M\frac{r^2}{2}$
Sphere, axis any diameter, r = radius	$I = \frac{2}{5}Mr^2$

TABLE IX.—*Boiling Point of Water Under Different Barometric Pressures.*

Barometric press.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
72.	98.49	98.53	98.57	98.61	98.65	98.69	98.72	98.76	98.80	98.84
73.	98.88	98.92	98.95	98.99	99.03	99.07	99.10	99.14	99.18	99.22
74.	99.26	99.29	99.33	99.37	99.41	99.44	99.48	99.52	99.55	99.59
75.	99.63	99.67	99.70	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76.	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77.	100.37	100.40	100.44	100.48	100.52	100.55	100.59	100.63	100.67	100.70

TABLE X.—*Heat Constants.*

Substance	Cubical Expansion	Specific Heat	Boiling Point	Heat of Vaporization
				cal. / g
Alcohol	0.00110	0.58	78°.3	210
Ether	0.00163	0.54	34 .9	90
Mercury	0.000181	0.0332	356 .7	62
Turpentine	0.00094	0.42	159 .0	70
Water	0.00018	1	100 .0	539

Substance	Linear Expansion	Specific Heat	Melting Point	Heat of Fusion
				cal. / g
Aluminium	0.000023	0.22	658°	
Brass	0.000019	0.093	900	
Copper	0.000017	0.093	1084	
Glass	0.000008	0.19	1200	
Ice	0.000105	0.504	0	79.5
Iron	0.000012	0.11	1520	30
Lead	0.000029	0.031	327	6
Nickel	0.000013	0.11	1450	
Platinum	0.000009	0.17	1755	27
Quartz, fused	0.00000045	0.19	2000	
Silver	0.000019	0.056	961	21
Tin	0.000023	0.054	232	13
Zinc	0.000029	0.094	419	28

TABLE XI.—*Vapor Tension of Liquids.*

Temperature	Alcohol	Ether	Mercury	Water
°C.	cm.	cm.	cm.	cm.
—10	0.65	11.4	.....	.....
0	1.25	18.5	0.00004	0.46
10	2.40	28.8	0.00005	0.92
20	4.41	44.0	0.0001	1.75
30	7.84	64.5	0.0003	3.18
40	13.36	91.0	0.0006	5.49
50	22.00	127.5	0.0013	9.20
60	35.1	173	0.0026	14.89
70	54.1	229	0.0051	23.33
80	81.2	300	0.0092	35.54
90	119	384	0.0162	52.59
100	169	490	0.028	76.00
110	236	607	0.045	107.5
120	322	760	0.075	149.1

TABLE XII.—*Index of Refraction for Sodium Light.*

Dry Air, 0°C	1.0003	Glass,		Calcite,	
Ice.....	1.31	Light Crown..	1 515	Ordinary ray.....	1.659
Water, 15°C.	1.3336	Light Flint ...	1.609	Extraordinary ray	1.486
Water, 20°C.	1.3332	Heavy Flint..	1.75	Quartz,	
Alcohol	1.362	Carbon Bisul-		Ordinary ray....	1.554
Benzine	1.501	phide, 20°C...	1.628	Extraordinary ray	1.553

TABLE XIII.—*Wave Lengths of Lines in Solar Spectrum, in Air at 20° C, Pressure 76 cm.; $\mu = 0.001$ mm.*

Line	Element	Wave Length	Line	Element	Wave Length	Line	Element	Wave Length
		μ			μ			μ
A	Atm. O	0.7628	D_2	Na	0.58900	*e*	Fe	0.43836
a	Atm. O	0.7185	E_1	Fe, Ca	0.52703	*f*	H	0.43405
B	O	0.68701	b_1	Mg	0.51837	*G*	Fe, Ca	0.43079
C	H	0.65629	*c*	Fe	0.48576	*h*	H	0.41018
α	O	0.62781	*F*	H	0.48614	*H*	H, Ca	0.39685
D_1	Na	0.58960	*d*	Fe	0 46682	*K*	Ca	0.39337

Wave Lengths of Lines in the Flame Spectrum of Various Metals.

Element	Wave Length	Element	Wave Length
	μ		μ
K	0.768	Tl	0.535
Li	0.671	Sr	0.461
Na	0.589	K	0.404

TABLE XIV.—*Electrical Resistance of Metals.*

(a) Specific conductivity, referred to mercury.

Aluminium (soft)	32.35	Nickel (soft)	3.14
Copper (pure)	59	Platinum	14.4
Iron	9.75	Silver (soft)	62.6
Mercury	1	Tin	7

(b) Specific resistance, in ohms cm. at 18° C.

Substance	Specific Resistance	Temperature Coefficient
Aluminium	0.32	0.0036
Copper	0.17	0.0040
German Silver	3.0	0.0004
Iron	1.2	0.0005
Manganin	4.2	0.00001
Mercury	9.58 $\times 10^{-5}$	0.00092
Nickel	1.0	0.005
Platinum, pure	1.08	0.0036
Platinum, commercial	1.4	0.0025
Silver	0.16	0.0037

TABLE XV.—*Electrical Conductivity of Solutions at 18°C.*

(a) Specific conductivity, in $ohms^{-1}\ cm^{-1}$.

Concentration	NaCl	$CuSO_4$	$ZnSO_4$	$AgNO_3$
5%	0.0672	0.0189	0.0191	0.0256
10%	0.1211	0.0320	0.0321	0.0476
15%	0.1642	0.0421	0.0415	0.0683

(b) Equivalent conductivity.

Gramequivalents per Liter	NaCl	KCl	KNO_3	$\frac{1}{2}H_2SO_4$
0.01	0.1028	0.1224	0.1182	0.308
0.1	0.0925	0.1120	0.1048	0.225
0.5	0.0809	0.1024	0.0892	0.205
1.0	0.0743	0.0983	0.0805	0.198

TABLE XVI.—*Numbers Frequently Required.*

1 cm. = 0.3937 in.
1 mile = 1.6093 km.
$\pi = 3.14159$.
$\pi^2 = 9.8696$.

1 in. = 2.540 cm.
1 km. = 0.6214 mile.
$\log \pi = 0.49715$.
$\log \pi^2 = 0.99430$.

Base of natural logarithms: $e = 2.7183$.
$\log e = 0.43429$.
Factor to convert common into Naperian logs = 2.3026.

Density of dry air at 0°C. under a barometric pressure of 76 cms., 0.001293 gm/cm³.
Coefficient of expansion of air . . . 0.00367.
Velocity of sound in air at 0°C. . . 332.4 m/sec.
1 calorie = 4.181×10^7 ergs for water at 20°C.
1 atmo. pressure = 1.0132×10^6 dynes/cm².
g at latitude 45° and sea level = 980.63 cm/sec².
Specific heat of steam at constant pressure . 0.4776.
Specific heat of steam at constant volume . 0.3637.
Vibration frequency of C_3 on the scientific diatonic scale, 256.
Vibration frequency of A_3 on the musical equal-tempered scale, 435.

TABLE XVII.—*Squares, Cubes, Square Roots, Circumferences and Areas of Circles.*

n	πn	$\frac{1}{4}\pi n^2$	n^2	n^3	$\sqrt{n}$
1	3,1416	0,7854	1	1	1,0000
2	6,2832	3,1416	4	8	4142
3	9,4248	7,0686	9	27	7321
4	12,566	12,566	16	64	2,0000
5	15,708	19,635	25	125	2361
6	18,850	28,274	36	216	4495
7	21,991	38,485	49	343	6458
8	25,133	50,265	64	512	8284
9	28,274	63,617	81	729	3,0000
10	31,416	78,540	100	1000	1623
11	34,557	95,03	121	1331	3166
12	37,699	113,10	144	1728	4641
13	40,841	132,73	169	2197	6056
14	43,982	153,94	196	2744	7417
15	47,124	170,17	225	3375	8730
16	50,265	201,06	256	4096	4,0000
17	53,407	226,98	289	4913	1231
18	56,549	254,47	324	5832	2426
19	59,690	283,53	361	6859	3589
20	62,832	314,16	400	8000	4721
21	65,973	346,36	441	9261	5826
22	69,115	380,13	484	10648	6904
23	72,257	415,48	529	12167	7958
24	75,398	452,39	576	13824	8990
25	78,540	490,87	625	15625	5,0000
26	81,68	530,93	676	17576	099
27	84,82	572,55	729	19683	196
28	87,96	615,75	784	21952	291
29	91,11	660,52	841	24389	385
30	94,25	706,86	900	27000	477
31	97,39	754,77	961	29791	568
32	100,53	804,25	1024	32768	657
33	103,67	855,30	1089	35937	745
34	106,81	907,92	1156	39304	831
35	109,96	962,11	1225	42875	916
36	113,10	1017,9	1296	46656	6,000
37	116,24	1075,2	1369	50653	083
38	119,38	1134,1	1444	54872	164
39	122,52	1194,6	1521	59319	245
40	125,66	1256,6	1600	64000	325
41	128,81	1320,3	1681	68921	403
42	131,95	1385,4	1764	74088	481
43	135,09	1452,2	1849	79507	557
44	138,23	1520,5	1936	85184	633
45	141,37	1590,4	2025	91125	708
46	144,51	1661,9	2116	97336	782
47	147,65	1734,9	2209	103823	856
48	150,80	1809,6	2304	110592	928
49	153,94	1885,7	2401	117649	7,000
50	157,08	1963,5	2500	125000	071

TABLE XVII.—*Continued.—Squares, Cubes, Square Roots, Circumferences and Areas of Circles.*

n	$\pi\, n$	$\frac{1}{4}\pi\, n^2$	n^2	n^3	$\sqrt{n}$
51	160,22	2042,8	2601	132651	7,141
52	163,36	2123,7	2704	140608	211
53	166,50	2206,2	2809	148877	280
54	169,65	2290,2	2916	157464	348
55	172,79	2375,8	3025	166375	416
56	175,93	2463,0	3136	175616	483
57	179,07	2551,8	3249	185193	550
58	182,21	2642,1	3364	195112	616
59	185,35	2734,0	3481	205379	681
60	188,50	2827,4	3600	216000	746
61	191,64	2922,5	3721	226981	810
62	194,78	3019,1	3844	238328	874
63	197,92	3117,2	3969	250047	937
64	201,06	3217,0	4096	262144	8,000
65	204,20	3318,3	4225	274625	062
66	207,35	3421,2	4356	287496	124
67	210,49	3525,7	4489	300763	185
68	213,63	3631,7	4624	314432	246
69	216,77	3739,3	4761	328509	307
70	219,91	3848,5	4900	343000	367
71	223,05	3959,2	5041	357911	426
72	226,19	4071,5	5184	373248	485
73	229,34	4185,4	5329	389017	544
74	232,48	4300,8	5476	405224	602
75	235,62	4417,9	5625	421875	660
76	238,76	4536,5	5776	438976	718
77	241,90	4656,6	5929	456533	775
78	245,04	4778,4	6084	474552	832
79	248,19	4901,7	6241	493039	888
80	251,33	5026,6	6400	512000	944
81	254,47	5153,0	6561	531441	9,000
82	257,61	5281,0	6724	551368	055
83	260,75	5410,6	6889	571787	110
84	263,89	5541,8	7056	592704	165
85	267,04	5674,5	7225	614125	220
86	270,18	5808,8	7396	636056	274
87	273,32	5944,7	7569	658503	327
88	276,46	6082,1	7744	681472	381
89	279,60	6221,1	7921	704969	434
90	282,74	6361,7	8100	729000	487
91	285,88	6503,9	8281	753571	539
92	289,03	6647,6	8464	778688	592
93	292,17	6792,9	8649	804357	644
94	295,31	6939,8	8836	830584	695
95	298,45	7088,2	9025	857375	747
96	301,59	7238,2	9216	884736	798
97	304,73	7389,8	9409	912673	849
98	307,88	7543,0	9604	941192	899
99	311,02	7697,7	9801	970299	950
100	314,16	7854,0	10000	1000000	10,000

TABLE XVIII.—*Trigonometric Functions.*

	Arc	Sine	Tangent	log arc	log sin	log tan
1°	0,0175	0,0175	0,0175	$\bar{2}$,2419	$\bar{2}$,2419	$\bar{2}$,2419
2	0349	0349	0349	5428	5428	5431
3	0524	0523	0524	7190	7188	7194
4	.0698	0698	0699	8439	8436	8446
5	0873	0872	0875	$\bar{1}$,9408	$\bar{1}$,9403	$\bar{1}$,9420
6	1047	1045	1051	$\bar{1}$,0200	$\bar{1}$,0192	$\bar{1}$,0216
7	1222	1219	1228	0870	0859	0891
8	1396	1392	1405	1450	1436	1478
9	1571	1564	1584	1961	1943	1997
10	1745	1736	1763	2419	2397	2463
11	1920	1908	1944	2833	2806	2887
12	2094	2079	2126	3210	3179	3275
13	2269	2250	2309	3558	3521	3634
14	2443	2419	2493	3879	3837	3968
15	2618	2588	2679	4180	4130	4281
16	2793	2756	2867	4461	4403	4575
17	2967	2924	3057	4723	4659	4853
18	3142	3090	3249	4972	4900	5118
19	3316	3256	3443	5206	5126	5370
20	3491	3420	3640	5429	5341	5611
21	3665	3584	3839	5641	5543	5842
22	3840	3746	4040	5843	5736	6064
23	4014	3907	4245	6036	5919	6279
24	4189	4067	4452	6221	6093	6486
25	4363	4226	4663	6398	6259	6687
26	4538	4384	4877	6569	6418	6882
27	4712	4540	5095	6732	6570	7072
28	4887	4695	5317	6890	6716	7257
29	5061	4848	5543	7042	6856	7438
30	5236	5000	5774	7190	6990	7614
31	5411	5150	6009	7333	7118	7788
32	5585	5299	6249	7470	7242	7958
33	5760	5446	6494	7604	7361	8125
34	5934	5592	6745	7733	7476	8290
35	6109	5736	7002	7860	7586	8452
36	6283	5878	7265	7982	7692	8613
37	6458	6018	7536	8101	7795	8771
38	6632	6157	7813	8216	7893	8928
39	6807	6293	8098	8330	7989	9084
40	6981	6428	8391	8439	8081	9238
41	7156	6561	8693	8546	8169	9392
42	7330	6691	9004	8651	8255	9544
43	7505	6820	9325	8753	8338	9697
44	7679	6947	9657	8853	8418	9848
45	7854	7071	1,0000	8951	8495	0,0000

TABLE XVIII.—*Continued.—Trigonometric Functions.*

	Arc	Sine	Tangent	log arc	log sin	log tan
46°	0,8029	0,7193	1,0355	$\bar{1}$,9047	$\bar{1}$,8569	0,0152
47	8203	7314	0724	9140	8641	0303
48	8378	7431	1106	9231	8711	0456
49	8552	7547	1504	9321	8778	0608
50	8727	7660	1918	9409	8843	0762
51	8901	7771	2349	9494	8905	0916
52	9076	7880	2799	9579	8965	1072
53	9250	7986	3270	9661	9023	1229
54	9425	8090	3764	9743	9080	1387
55	9599	8192	4281	9822	9134	1548
56	9774	8290	4826	9901	9186	1710
57	9948	8387	5399	9977	9236	1875
58	1,0123	8480	6003	0,0053	9284	2042
59	0297	8572	6643	0127	9331	2212
60	0472	8660	7321	0200	9375	2386
61	0647	8746	8040	0272	9418	2562
62	0821	8829	8807	0343	9459	2743
63	0996	8910	9626	0412	9499	2928
64	1170	8988	2,0503	0480	9537	3118
65	1345	9063	1445	0548	9573	3313
66	1519	9135	2460	0614	9607	3514
67	1694	9205	3559	0680	9640	3721
68	1868	9272	4751	0744	9672	3936
69	2043	9336	6051	0807	9702	4158
70	2217	9397	7475	0870	9730	4389
71	2392	9455	9042	0931	9757	4630
72	2566	9511	3,0777	0992	9782	4882
73	2741	9563	2709	1052	9806	5147
74	2915	9613	4874	1111	9828	5425
75	3090	9659	7321	1169	9849	5719
76	3265	9703	4,0108	1227	9869	6032
77	3439	9744	3315	1284	9887	6366
78	3614	9781	7046	1340	9904	6725
79	3788	9816	5,1446	1395	9919	7113
80	3963	9848	6713	1450	9934	7537
81	4137	9877	6,3138	1504	9946	8003
82	4312	9903	7,1154	1557	9958	8522
83	4486	9925	8,1443	1609	9968	9109
84	4661	9945	9,5144	1662	9976	9784
85	4835	9962	11,4301	1713	9983	1,0580
86	5010	9976	14,3007	1764	9989	1554
87	5184	9986	19,0811	1814	9994	2806
88	5359	9994	28,6363	1864	9997	4569
89	5533	9998	57,2900	1913	9999	7581
90	5708	1,0000	∞	1961	0,0000	∞

TABLE XIX.—*Logarithms.*

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774

TABLE XIX.—*Continued.—Logarithms.*

N	0	1	2	3	4	5	6	7	8	9
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	00000	0043	0087	0130	0173	0217	0260	0303	0346	0389
101	00432	0475	0518	0561	0604	0647	0689	0732	0775	0817
102	00860	0903	0945	0988	1030	1072	1115	1157	1199	1242
103	01284	1326	1368	1410	1452	1494	1536	1578	1620	1662
104	01703	1745	1787	1828	1870	1912	1953	1995	2036	2078
105	02119	2160	2202	2243	2284	2325	2366	2407	2449	2490
106	02531	2572	2612	2653	2694	2735	2776	2816	2857	2898
107	02938	2979	3019	3060	3100	3141	3181	3222	3262	3302
108	03342	3383	3423	3463	3503	3543	3583	3623	3663	3703
109	03743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	4179	4218	4258	4297	4336	4376	4415	4454	4493

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